MATHEMATICS

A Textbook for Class VI

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राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

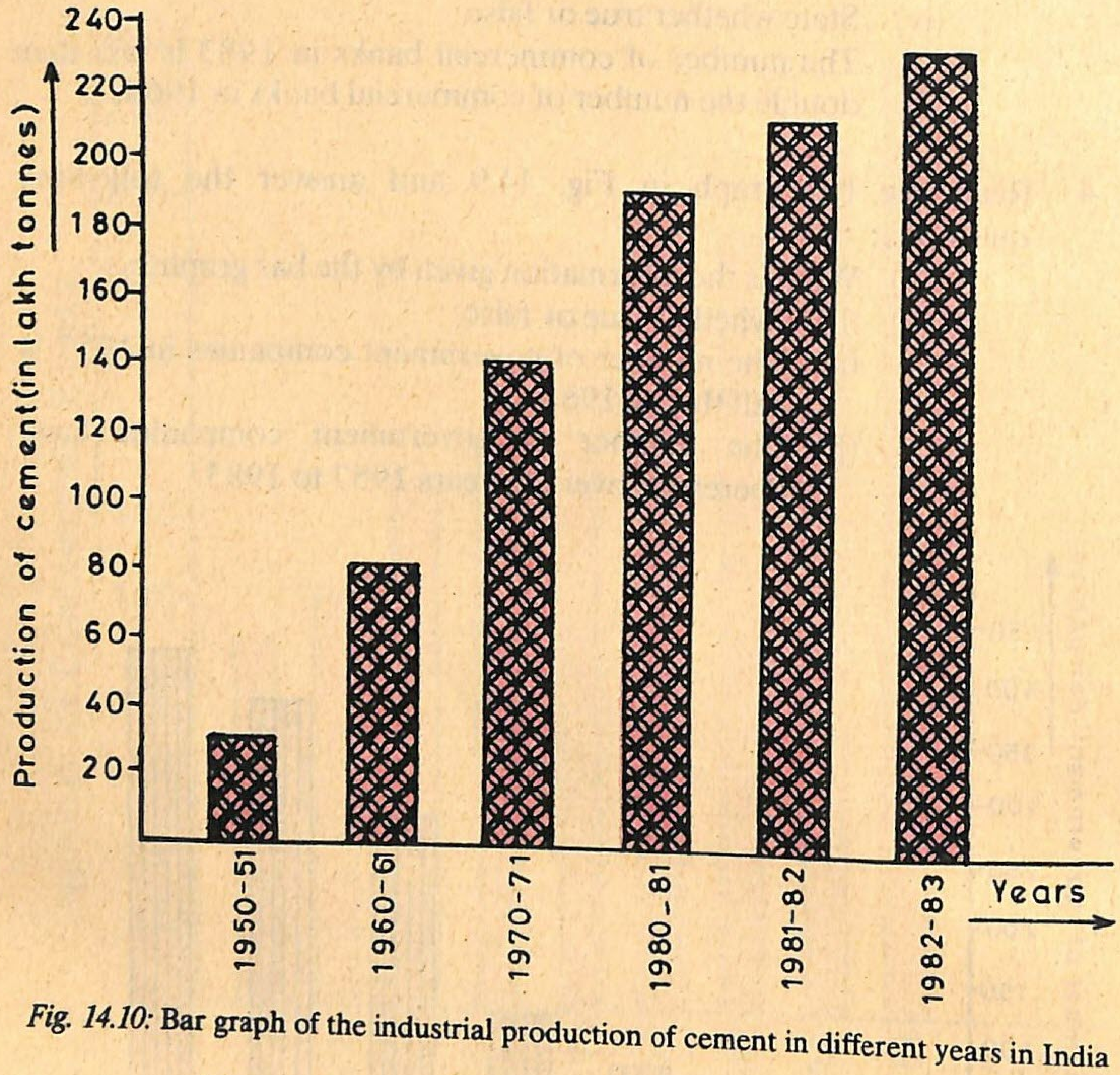

Fig. 14.10: Bar graph of the industrial production of cement in different years in India

(ii) What was the production of cement in the year 1980-81?

(iii) What is the minimum and the maximum productions of cement and the corresponding years?

6. Read the bar graph in Fig. 14.11 and answer the following questions:

   (i) What information is given by the bar graph?

   (ii) What was the crop production of rice in 1970-71?

(iii) What is the difference between the maximum and the minimum productions of rice?

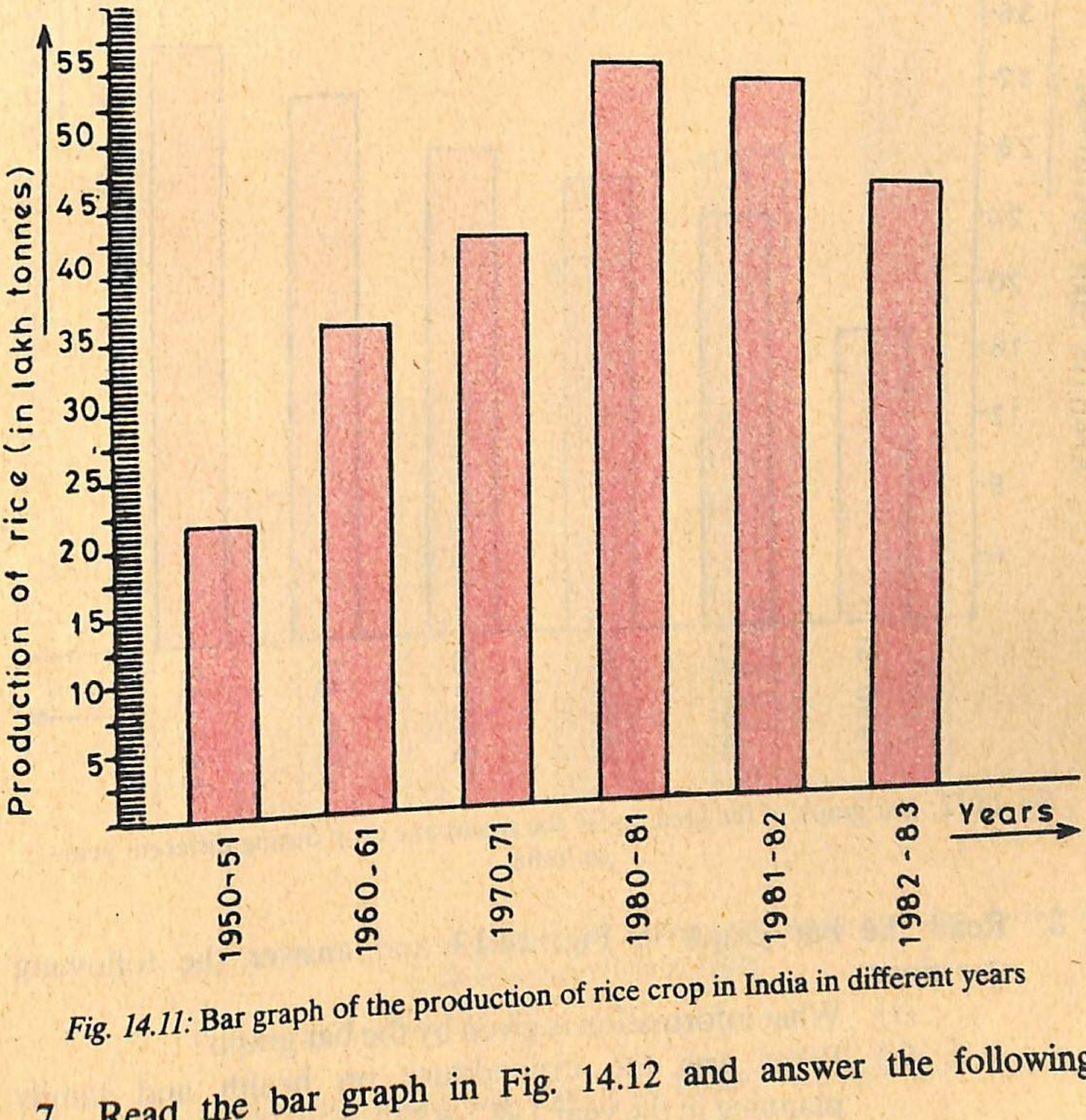

Fig. 14.11: Bar graph of the production of rice crop in India in different years

7. Read the bar graph in Fig. 14.12 and answer the following questions:
   (i) What information is given by the bar graph?
   (ii) In which years the areas under the sugarcane crop were the maximum and the minimum?

June 1987
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Foreword

MATHEMATICS occupies a key place in the curriculum of our educational system in the present age of automation and computers. Its dimensions and domains are relevant to and cut across almost all disciplines. Therefore, it is absolutely necessary these days to acquire an adequate knowledge of mathematics. Our National Policy on Education (NPE) 1986 has laid a greater emphasis on the education of mathematics right from the school stage.

The Department of Education in Science and Mathematics (DESM) has redesigned the mathematics curriculum in the light of the NPE 1986. The basic guiding principle in this task is that the teaching of mathematics should take into account the environment of the pupil as well as the relevant needs of our society.

The present book has been developed according to the revised curriculum by the DESM to meet the objectives of the NPE 1986. The book aims at the development of learning by inference from specific examples to help understand the basic principles, operations and processes of elementary mathematics. A simple and lucid language within the scope of the pupil's vocabulary at this level has been used and unnecessary difficult terminology avoided. The concepts have been developed through concrete examples from the pupil's environment.

The detailed planning and coordination for the development of this book was done by Dr. Surja Kumari with the assistance of Shri Ishwar Chandra and Shri Mahendra Shanker as members of the project team of the DESM.

The draft materials were developed in a workshop held at the NIE in September 1986, and revised and finalized by Dr. Surja Kumari, Shri Ishwar Chandra and Shri Mahendra Shanker. This draft was reviewed in a workshop held at the NIE in November 1986. The final draft was reviewed by Prof. D.K. Sinha, Pro Vice-Chancellor, Calcutta University, in a vetting group meeting held at Calcutta in January/February 1987. His valuable suggestions, additions and deletions of some units of the curriculum were incorporated in the manuscript, which was finalized for the press.

I record my sincere appreciation of the efforts put in by Prof. D.K. Sinha for the final review of the manuscript. I also record my grateful thanks to Dr. Surja Kumari, Shri Ishwar Chandra and Shri Mahendra Shanker for their dedication and deligence in completing the task assigned to them within the stipulated short period. I am grateful to Prof. A.K. Jalaluddin, Joint Director, NCERT, for his valuable guidance and to Prof. B. Ganguly for taking keen interest in the development of this book. I am also grateful to other members of the Mathematics Group—Prof. K.V. Rao, Prof. S.C. Das, Dr. B. Deokinandan and Dr. Ram Autar—for their valuable suggestions and help during the various stages of development of this book.

The Council will be grateful to students, teachers and other users of this book for their reactions and suggestions for improving the book in subsequent editions.

P. L. MALHOTRA
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Preface

THE National Council of Educational Research and Training has drawn up a revised curriculum of mathematics in the light of the National Policy on Education 1986. This curriculum differs from the previous one not only in the specific units to be covered in the various classes but also in the emphasis it lays on the minimum level of learning at each stage. The present textbook has been developed according to the revised curriculum.

This textbook contains five units divided into fourteen chapters. Each unit consists of a brief historical note pertaining to various topics covered in that unit, highlighting the contributions of Indian mathematicians. Pupils are advised to collect some more information about these mathematicians and display their contributions in the form of charts. Each chapter ends with a list of things to be remembered by heart so that these can be readily applied to different problem-solving situations.

The book contains a number of suggested activities for better and deeper understanding of concepts. Attempts have been made to explain every concept in detail so that the motivated pupil can study on his own and be ahead of the class. Each chapter contains stimulating exercises including those from the application of mathematics to different fields such as commerce etc. A number of word problems have been included, which help the pupil to inculcate the national values, namely, small family norms, the equality of sex, equal opportunities to every caste or creeds etc.

The major emphasis of this book is on understanding basic mathematical ideas which are sometimes obscured by giving stress on the teaching of computational techniques and on acquiring and performing these skills mechanically. The emphasis in this book is not only on the 'how' but on the 'why' of the mathematical operations also.

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UNIT ONE

ARITHMETIC

IN OUR DAILY LIFE, we use numbers on different occasions and situations. For instance, Sarita gets up at 6 a.m., her school bus number is 22, she studies in Class 6, her annual tuition fee is Rs 75, she has 12 notebooks, etc.—these are all examples of uses of number what we call counting. Man in ancient days did not know counting but had to keep a record of his personal belongings like cattle, trees, etc., and for this he had to rely on markings in the form of notches on a tally stick or knots in a string. In a way, what he used to do, was to set up a one-to-one correspondence between the markings and his belongings and thereby he could keep a count of them.

Sometimes later, a need was felt to invent a better way of keeping records and this gave rise to the search for numbers. That is how various civilizations developed their own systems, what we call numeration.

The Egyptian system of numeration is found in carvings on tombs and monuments. It is almost 5,000 years old. It uses a decimal system similar to that of Romans with separate symbols for powers of 10; 10 is represented by a heel bone ∩, 100 by a scroll ᘓ, etc.

Romans used a system with special signs for larger numbers but without the notion of place values—I stood for 1, V for 5, X for 10, L for 50, C for 100, D for 500 and M for 1,000. A smaller symbol on the left of a higher one represented subtraction and on the right, addition. This system also did not gain favour as it was found inconvenient in computations.

By 300 B.C., the ancient Indians had a set of numerals called *Brahm Sankhyayen* but they did not use place value and had no numeral for zero.

The Hindu astronomer Aryabhatta had a student Bhaskara who about A.D. 500 used a place value system that had a symbol for zero. The unit's place (one's place) was at the left instead of right today. This was soon changed and gradually the present *Hindu-Arabic system of numeration* developed which is most scientific and convenient in reading, writing and has been internationally accepted and used.

The Hindu-Arabic system of numeration, as the name suggests, was developed in India. The Arabs took it and made some modifications in its numerals. Europeans inherited these numerals in modified form from the Arabs.

This system uses ten symbols—0, 1, 2, 3, 4, 5, 6, 7, 8 and 9—called digits and any number, howsoever large, can be written with the help of only these symbols by means of the place value principle. This system has two unique features which make it superior to all other systems. First, it introduced the symbol for 'zero' to express the absence of a quantity which made possible to use the place value principle. Secondly, it permits the development of easy rules for performing the four fundamental operations. Since this system uses ten symbols and makes use of arrangements by tens in writing numerals for large numbers, this is also called *base-10* or *decimal system of numeration*.

In this unit we shall study natural (counting) numbers and whole numbers, their operations, properties of operations and extend the system of whole numbers to that of integers.

CHAPTER ONE

Review of Work done in Earlier Classes

IN THIS CHAPTER we revise some of the mathematical ideas learnt in earlier classes. Let us do this through problems in which we can make use of our previous knowledge.

EXERCISE 1

1. Which of the following stands for 'five lakh four thousand three'?
   50403   504003   540003
2. Write each of the following numbers in words:
   (i) 9905057   (ii) 60050060
3. Write each of the following numbers in expanded notation:
   (i) 675028   (ii) 730021
4. Counting by thousands write the numbers from
   (i) 7478125 to 7483125
   (ii) 15003216 to 15009216
5. Counting by fives write the numbers in reverse order from
   (i) 10000005 to 9999995
   (ii) 5050404 to 5050389
6. Find the difference of the place values of two fives in 576502.
7. Determine the difference between the largest and smallest 5-digit numbers.

8. In each of the following replace $*$ by $<$ or $>$ to make the statement true:
    - (i) $64231 * 321045$
    - (ii) $3000000 - 1 * 3999999$
    - (iii) $5009006 * 5006009$
9. Give the successor of 699999.
10. Give the predecessor of 4004000.
11. Rewrite in ascending order with proper symbol:
    30303030, 3000333, 33003300, 30030033
12. Rewrite in descending order with proper symbol:
    1100111, 1110011, 1101011, 1110202, 1111010
13. Find the value of
    - (i) 378954×889
    - (ii) 210459×0
    - (iii) 70810×98
    - (iv) $594725 + 550023 + 69120037 + 732981$
    - (v) $689324 \times 59 + 41 \times 689324$
    - (vi) $39568 \times 5 + 256935 + 5 - 3001$
14. Using the rules of divisibility state which of the following numbers are divisible by 2; by 5:
    (i) 22481 (ii) 15280 (iii) 69435
15. Using the rules of divisibility state which of the following numbers are divisible by 3; by 9:
    (i) 53421 (ii) 32879 (iii) 462753
16. Find the H.C.F. of each of the following sets of numbers:
    - (i) 77, 42, 63
    - (ii) 18, 42, 102
17. Find the L.C.M. of each of the following sets of numbers:
    - (i) 15, 20, 30
    - (ii) 66, 75, 130
18. The L.C.M. and H.C.F. of two numbers are 720 and 18, respectively. If one of the numbers is 90, determine the other.

19. In a morning walk, three girls step off together from the same spot. Their steps cover 33 cm, 36 cm and 42 cm, respectively. At what distance from the starting point will they again step off together?

20. For each of the following give the corresponding Hindu-Arabic numeral:
    (i) XIX (ii) XXII (iii) XXIV

21. Write the corresponding Roman numeral for
    (i) 13 (ii) 18 (iii) 29

22. Rewrite in increasing order with proper symbol:

$$\frac{5}{6}, \quad \frac{9}{24}, \quad \frac{3}{2}, \quad \frac{1}{3}, \quad \frac{5}{8}$$

23. Rewrite in decreasing order with proper symbol:

$$\frac{6}{7}, \quad \frac{7}{8}, \quad \frac{8}{9}, \quad \frac{9}{12}, \quad \frac{15}{18}$$

24. Find the value of each of the following in simplified form without converting fractions into decimals or vice versa:

(i) $\frac{1}{3} - \frac{5}{8} + \frac{3}{5}$

(ii) $6 - 2\frac{1}{3} + 3\frac{3}{4} - \frac{15}{13}$

(iii) $6 - 0.6 - 0.006 - 0.0006$

(iv) $71.4 + 16.6 - 32.08 + 1.005$

25. Give the result of adding the difference of 3.003 and 2.05 to their sum.

26. On a festive occasion, out of 21 litres of milk 17.3 litres were used in the preparation of sweets and 3.6 litres were used in the preparation of cheese. How much milk was left?

27. In school meals 50 kg of rice were consumed daily by students. If each student gets 40 g of rice, determine the number of students in the school.
28. Sheila went to a market with Rs 10. She purchased 6 notebooks each costing Rs 1.20 and two pencils each costing 85 paise. What amount of money was left with her?
29. A student starts for his school at 9.40 a.m. and comes back home at 4.15 p.m. Determine the time spent by him in the school if half an hour is spent in coming and going.
30. Convert each of the following fractions into a decimal:

    (i) $\frac{1}{25}$ (ii) $\frac{2}{5}$ (iii) $\frac{3}{4}$
31. Convert each of the following decimals into a common fraction:

    (i) 3.6 (ii) .50 (iii) 1.125
32. A train starts from a city on Thursday at 8 p.m. and reaches another city on Saturday at 8 a.m. If the distance between the two cities is 1692 km, determine the average speed of the train.
33. Determine the perimeter and area of a floor whose length and breadth are 18m and 15m, respectively.
34. Determine the perimeter and area of a square field with side 63 m.
35. Which of the following statements are true and which are false?
    - (i) In our system (decimal system) of numeration, there are, in all, 9000 4-digit numbers.
    - (ii) $.1 < .01$
    - (iii) $\frac{4}{5} < \frac{5}{6}$
    - (iv) The area of a rectangle remains the same if its length is doubled and breadth is halved.
    - (v) The smallest 4-digit number having exactly three different digits is 1012.
36. A boy saves 45 paise daily. Find the least number of days in which he will be able to save an exact number of rupees.

37. A drum is $\frac{2}{3}$ full. If 50 litres more are required to fill it up, determine the capacity of the drum.

38. In each of the following there are four alternative answers given. Point out the correct answer.

    (i) The product of 0.003 and 0.8 is
    (a) 0.24 (b) 0.024 (c) 0.0024 (d) 0.00024

    (ii) Which fraction is *not* equal to the other three?
    (a) $\frac{6}{10}$ (b) $\frac{3}{5}$ (c) $\frac{6}{9}$ (d) $\frac{9}{15}$

    (iii) A number is divisible by 18 if it is divisible by
    (a) 2 and 3 (b) 3 and 6 (c) 4 and 6 (d) 2 and 9

39. A snail climbs up a vertical stick 12 cm high. Each day it climbs 3 cm but each night slips 2 cm back. On what day does it reach the top of the stick?

40. In each of the following, observe the pattern and fill in the blanks:

(i)
$1 \times 9 + 2 = 11$
$12 \times 9 + 3 = 111$
$123 \times 9 + 4 = 1111$
$\_\_ \times 9 + \_ = \_\_$

(ii)
$1 = 1 \times 1$
$1 + 2 + 1 = 2 \times 2$
$1 + 2 + 3 + 2 + 1 = 3 \times 3$
$\_ + \_ + \_ + \_ + \_ + \_ + \_ = \_ \times \_$

(iii)
$1089 \times 1 = 1089$
$1089 \times 2 = 2178$
$1089 \times 3 = 3267$
$1089 \times 4 = \_\_$

(iv)
$11 \times 11 = 121$
$111 \times 111 = 12321$
$111 \times 1111 = 1234321$
$\_\_ + \_\_ = \_\_$

41. After Independence, India made tremendous progress in the field of education. Given below is some information about it:

	1950	1982
Number of middle schools	13,596	1,23,423
Number of high schools	7,288	52,279
Number of middle school teachers	85,496	8,56,399
Number of high school teachers	1,26,504	9,93,115
Number of pupils in Classes I-V	192 lakhs	770 lakhs
Number of pupils in Classes VI-VIII	31 lakhs	221 lakhs
Number of pupils in Class IX	12 lakhs	118 lakhs

Now, fill in the blanks in each of the following:

(i) Total number of schools in 1950 was ______

(ii) The increase in the total number of teachers from 1950 to 1982 was ______.

(iii) The number of pupils in Classes I to V is increased by more than ______ times.

(iv) The increase in total number of pupils from Classes I to IX from 1950 to 1982 was ______

42. In 1981 Uttar Pradesh had 5,88,19,276 males and 5,20,42,737 females. Of these 2,27,98,451 males and 73,06,809 females were literate.

(a) What was the

  (i) total population of Uttar Pradesh in 1981?

  (ii) total number of literates?

  (iii) total number of illiterates?

(b) Which of the following statements are true?

  (i) More than half of the males were illiterate.

  (ii) Less than one-sixth of the females were literate.

  (iii) About one-fourth of the population was literate.

CHAPTER TWO

Natural Numbers and Whole Numbers

2.1 Natural Numbers

WHEN WE LOOK at a particular collection of objects, an idea of 'how many in the collection', comes to our mind. For example, when we look at each of the following collections of objects, one common property that comes to our mind is the number 'three'.

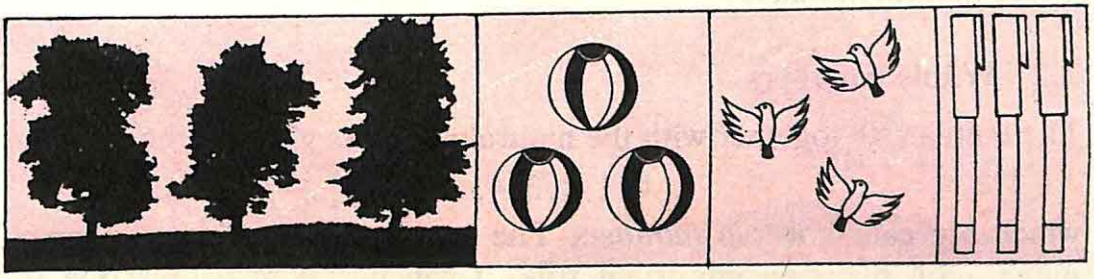

Fig. 2.1

We express this idea of number 'three' by writing the numeral '3'. In the same way we express the idea of number 'five' by writing numeral '5', and so on.

Different civilizations use different symbols (numerals) to denote the same number. The corresponding numerals used by some civilizations to denote first ten numbers and hundred are given below:

Hindu-Arabic	1	2	3	4	5	6	7	8	9	10	100
Roman	I	II	III	IV	V	VI	VII	VIII	IX	X	C
Egyptian	I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII	∩	𓍢
East-Arabic	I	۲	۳	۴	۵	۶	<	>	۹	10	100

We thus see that a number related to the objects in a collection gives an idea of how many objects are there in the collection.

The numeral is the symbol to denote that number. However, we shall not distinguish numbers from numerals hereafter.

In our daily life we come across statements like

(i) There are 12 months in a year.
(ii) There are 26 letters in English alphabet.
(iii) There are 7 days in a week.

Each of the numbers 12, 26, 7 in the statements mentioned above denotes the number of objects in a particular collection. For example, in the first case the collection is 'year' and objects are 'months' and 12 denotes the number of months. Similarly, in some other collections the number of objects may be one, two, three, etc. We have already studied these numbers, i.e. 1, 2, 3, 4, 5, . . . in our earlier classes. These are called *natural numbers.*

2.2 Whole Numbers

The number '0' together with the natural numbers gives us the numbers

$$0, 1, 2, 3, 4, \ldots$$

which are called *whole numbers.* The number zero in our system of numeration plays an important role. It can be taken to describe the number of elements in a collection with no objects.

In our system of numeration, any number whatsoever can be written with the help of symbols 0, 1, . . . 9, called digits.

Thus 0, 1, . . . 9 in our system serve the following two purposes:

(i) Each of them represents a number, specifying the number of objects in a collection. For example, the number '0' specifies no object in a collection, 1 specifies one object in a collection, and so on.
(ii) In a numeral, each of them works as a place holder. If '9' occurs in hundred's place, it represents 9 hundred, i.e. 900; if '0' occurs in ten's place, it represents 0 ten; 2 occurring in thousand's place represents 2000, and so on.

2.3 Base and Place Value

When we speak of base 10, we simply mean that we are thinking of collections (groupings) by tens. That is, given a collection of objects, we might ask 'how many groups of tens can be formed'. Let us, for example, consider the collection of dots in Fig. 2.2.

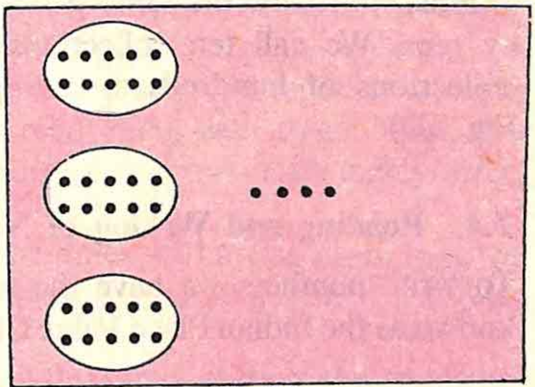

Fig. 2.2

We see that there are three groups of tens and 4 dots are left over. Instead of writing '3 groups of tens and 4 more', we write 34. The digit 3 in the second place from the right, called ten's place, represents 3 groups of 10, i.e. 30.

When we work with larger numbers, we need to have the collections

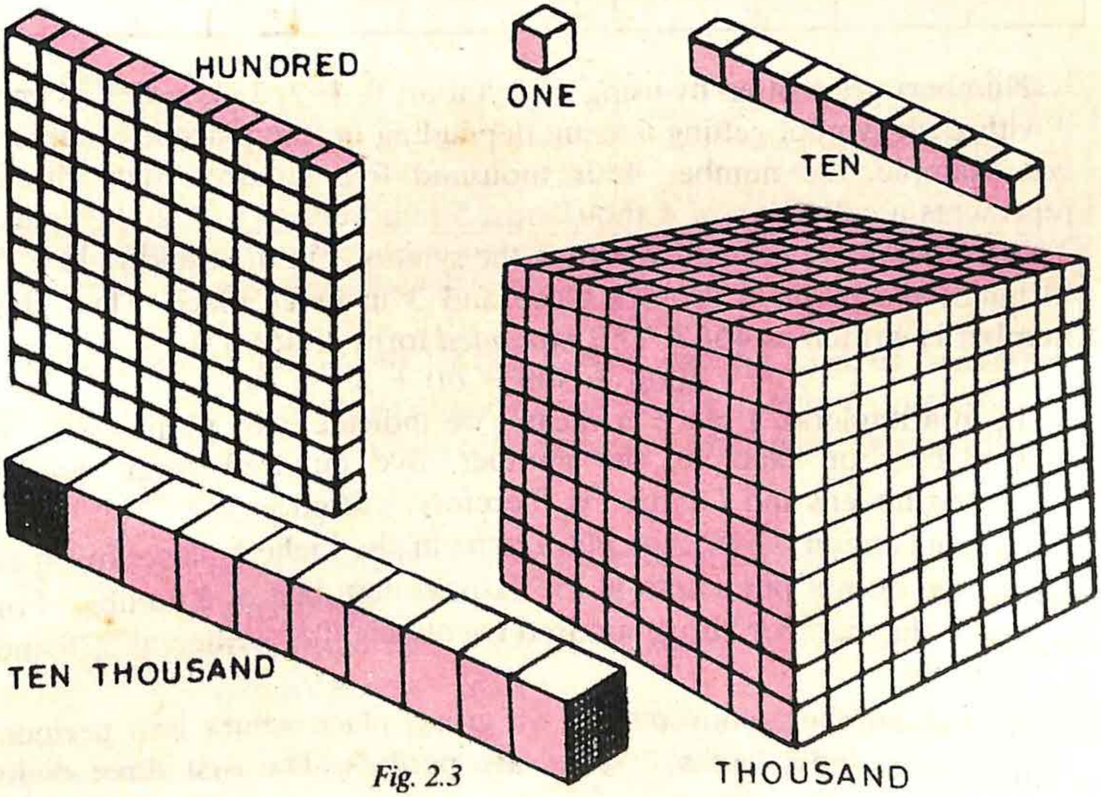

Fig. 2.3

of tens, we follow the same pattern and group these collections of tens by tens. We call ten collections of tens as a hundred. We call ten collections of hundreds as a thousand. This process continues (see Fig. 2.3).

2.4. Reading and Writing of Numbers

To write numbers we have the following place value chart popularly known as the Indian Place Value Chart.

Periods →	Crores		Lakhs		Thousands				
Places →	Ten crore	Crore	Ten lakh	Lakh	Ten thousand	Thousand	Hundred	Ten	Unit
	100000000	10000000	1000000	100000	10000	1000	100	10	1

Numbers are written by using the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 with each symbol getting a value depending on the place it occupies. For example, the number 'four thousand five hundred sixty three' represents a collection of 4 thousands, 5 hundreds, 6 tens and 3 units. This is, therefore, written by putting the symbol 4 in thousand's place, 5 in hundred's place, 6 in ten's place and 3 in unit's place. Thus, this number is written as 4563. The *expanded* form of 4563 is

$$4000 + 500 + 60 + 3.$$

If, in a numeral, a place is vacant, we indicate it by putting a '0' in that place. For example, the number 'five hundred seven' has 5 hundreds, no tens and 7 units; it is, therefore, written as 507.

By convention we do not put a zero in the highest place. In other words, we do not put a zero at the extreme left digit of a number. For example, the number 'three hundred twenty-eight' is written as 328 and not as 0328.

To read numbers conveniently we group place values into periods. 'Units', 'thousands', 'lakhs', 'crores' are periods. The first three digits from the right of a numeral make unit's period. The next two make thousand's period, the next two as lakh's period and the next two as crore's period. The digits in the same period are read together and the name of the period (except units) is read along with them. Thus, the number 17283546 is read as 'one crore seventy-two lakh eighty-three thousand five hundred forty-six'.

When the numerals are large, sometimes to have a clear look of periods, we insert commas after each period. Thus, we write 17283546 as 1,72,83,546.

The place value principle remaining the same, most of the countries use the place value chart which has different names of periods and places beyond thousand's place. Instead of lakhs, crores, etc. as periods, they use millions, billions, etc. Given below is the place value chart which is being followed by most of the countries of the world and is, therefore, referred to as the *international system of numeration.*

Periods→	* Billions		Millions			Thousands					
Places→	Ten billion	Billion	Hundred million	Ten million	Million	Hundred Thousand	Ten Thousand	Thousand	Hundred	Ten	Unit
	10000000000	1000000000	100000000	10000000	1000000	100000	10000	1000	100	10	1

The number 17283546 in this system will be read as 'seventeen milion two-hundred-eighty-three thousand five hundred forty-six'. To have a clear look of periods in this system, we shall write this number as 17,283,546.

Example 1: Determine the difference of the place values of two 8's in 278564083.

Solution: The place value of 8 in the second place from right, i.e. in ten's place = 80.

---

* In the British system of numeration, a billion equals one million millions.

Similarly, the place value of other 8 = 8000000.
The required difference = 8000000 − 80
= 7999920

Example 2: How many 5-digit numbers in all are there in base-10 system?

Solution: The largest 5-digit number is 99999. The largest 4-digit number is 9999.

Therefore, the total number of 5-digit numbers
= 99999 − 9999
= 90000

Example 3: Using all the digits 2, 3 and 5 only once write six different natural numbers.

Solution: We first put 2 in unit's place and make two natural numbers (i) 352 by putting 5 and 3 in ten's and hundred's places, respectively, and (ii) 532 by putting 3 and 5 in ten's and hundred's places, respectively.

Next, we put 3 in unit's place and get two more natural numbers 253 and 523.

Similarly, by putting 5 in unit's place, we get two other numbers 235 and 325. Therefore, the required six natural numbers are

352, 532, 253, 523, 235, 325

Example 4: Write the smallest 6-digit natural number having three different digits.

Solution: Three smallest digits are 0, 1 and 2. We know that we cannot put 0 (the smallest digit) as the left-most digit of a number. Therefore, we put the next smallest digit, i.e. 1 in the left-most place (the highest place) of the number.

We put the largest of these digits, i.e. 2 in unit's place which is the smallest place in a number and fill all other places by zeros. We thus get the required number as 100002.

EXERCISE 2.1

1. Write down (i) the smallest whole number, and (ii) the smallest natural number. Can you write the largest whole number?
2. Find the place value of the digit 4 in each of the following:
   (i) 24581370 (ii) 2043876
3. Determine the place value of 1 in 70100.
4. Find the product of the place values of two fives in 350256.
5. Find the difference of the place value and face value* of the digit 2 in 3124698.
6. How many numbers in all do we have with
   (i) 4 digits ? (ii) 6 digits ?
7. How many times does the digit 1 occur in ten's place in the numbers from 100 to 1000 ?
8. Write each of the following in expanded notation:
   (i) 3057 (ii) 235060
9. Write the corresponding number for each of the following:
   (i) $4 \times 10000000 + 3 \times 1000000 + 5 \times 10000$
   $6 \times 1000 + 1 \times 100 + \mathrm{v}\ 1 \times 10 + 9$
   (ii) $7000000 + 80000 + 70 + 7$
10. How many different 3-digit numbers can be formed by using the digits 0, 2, 5 without repeating any digit in the number?
11. Write the smallest 3-digit number which does not change if the digits are written in reverse order.
12. How many thousands make a million?
13. How many millions make a billion?
14. Write each of the following numbers in words in the international system of numeration:
    (i) 5342716 (ii) 10023078
15. Write in words the following facts in the Indian system of numeration:
    (i) Distance of the sun from the earth:
    148800000 km approximately

---

*Face value of a digit is the digit itself.

(ii) Population of India:
In 1971 : 546955945
In 1981 : 683810051

16. Write 10075302 in words and rearrange the digits to get the smallest number.
17. Write the greatest 7-digit number having three different digits.
18. Write the smallest 7-digit number having four different digits.

2.5 Representation of Whole Numbers on a Number Line

In order to discover more properties of whole numbers we need to represent them on a line called Number Line. We draw a straight line and mark a point 'O' on it corresponding to the number 'zero'. Starting from O,

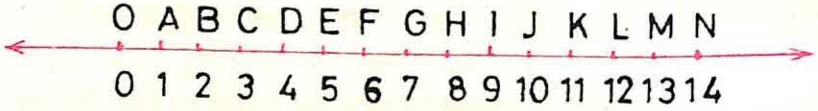

Fig. 2.4

we mark on it the points A, B, C, D, etc. at equal steps (distances) to the right of O in order (Fig 2.4). If we consider OA as 1 unit, then each of AB, BC, CD, DE, etc. represents 1 unit.

Thus,

OA = 1 unit
OB = OA + AB = 1 + 1, i.e. 2 units
OC = 3 units
OD = 4 units

and so on. Since O corresponds to the whole number 0, A, B, C, D, etc. correspond to the whole numbers 1, 2, 3, 4, etc., respectively. So, on the number line against A, B, C, D, etc. we write 1, 2, 3, 4, etc., respectively (see Fig. 2.4). In Fig. 2.4, we note that N (14) is not the last point of the number line. We can go up to any number as we like on the right by going along the line as long as we need. The arrow marks on both ends of the line indicate that the number line extends indefinitely on both sides. We have, thus, represented whole numbers on the number line. Let us observe the number line drawn above and derive the following important properties of whole numbers:

Property I: There is no whole number on the left of '0' and every number on its right is greater than it. Therefore, '0' is the smallest whole number. There is no largest whole number.

Property II: We note that 'one' is one more than 'zero', 'two' is one more than 'one', 'three' is one more than 'two', and so on. One is called the *successor* of zero, two, the successor of one, three that of two, and so on.

We conclude that *each whole number has one and only one successor* and it is the successor of the whole number just on its left on the number line. We note that 0 (zero) is not the successor of any whole number.

Property III: In the reverse way we note that 'zero' is one less than 'one', 'one' is one less than 'two', 'two' is one less than 'three', and so on. 'Zero' is called the *predecessor* of 'one', 'one' the predecessor of 'two', 'two' that of 'three', and so on. We conclude that *each whole number (except zero) has one and only one predecessor* and it is the predecessor of the number just on its right on the number line. The number zero has no predecessor in whole numbers.

Property IV: Every whole number on the number line is greater than every whole number on its left. For example,

$$14 > 8, \qquad 12 > 10, \qquad 8 > 7$$

Property V: Every whole number on the number line is less than every whole number on its right. For example,

$$9 < 11, \qquad 2 < 6, \qquad 11 < 14$$

Representation of whole numbers on the number line, therefore, helps us to compare two whole numbers, i.e. to decide which of the two is greater or smaller. However, sometimes it is not practicable to first represent two given whole numbers on a number line and then see their relative positions to decide which of them is greater. Let us, for example, consider the numbers 98 and 176. To compare these numbers also, the number line gives a hint. We know that $2 < 6$ implies that we need a whole number to be added to 2 to make it 6. In this case it is 4 since $2 + 4 = 6$; similarly, $3 < 5$ since $3 + 2 = 5$. On the same lines we can deduce that $98 < 176$ since $98 + 78 = 176$. Therefore, with the help of the number line we derive an important property of whole numbers, called the *property* of order of whole numbers.

Property VI: Of the given two different whole numbers, the smaller is that to which we have to add a natural number so that the sum equals the other.

The number line also displays the property of *betweenness.* For example, 4 lies between 2 and 6 since $2 < 4$ and $4 < 6$. We express this by writing $2 < 4 < 6$.

Example 5: Which whole number is the successor of 99?

Solution: We know that the next whole number on the right is the successor of a given whole number. Thus the successor of a whole number is 1 more than the number.

Therefore, the successor of 99 $= 99 + 1$

$= 100$

EXERCISE 2.2

1. Give the successor of each of the following:
   (i) 1000909 (ii) 2340700 (iii) 1039999
2. Write down the predecessor of each of the following:
   (i) 93 (ii) 2000 (iii) 7008000
3. How many whole numbers are there between 31 and 51?
4. Are all natural numbers also whole numbers? Are all whole numbers also natural numbers?

5. In each of the following pairs of numbers state which number is greater:
    (i) 53, 503 (ii) 307, 307
    (iii) 9876, 6789 (iv) 9830415, 100023001
6. Write down the next three consecutive whole numbers starting from 1009998.
7. Write down the three consecutive whole numbers just preceding 8510001.
8. 'There does not exist the largest natural number'. Give arguments in support of this statement.
9. Which of the following statements are true?
    (i) The smallest 6-digit number ending in 5 is 102345.
    (ii) The smallest 5-digit number is the successor of the largest 4-digit number.
    (iii) Of the given two natural numbers, the one having more digits is greater.

Things to Remember

1. The smallest natural number is 1 and the smallest whole number is 0.
2. The successor of a whole number is 1 more than the whole number.
3. The predecessor of a whole number is 1 less than the whole number. There is no predecessor of zero in whole numbers.
4. There do not exist the largest natural number and the largest whole number.
5. Our system of numeration is called Hindu-Arabic, base-10 or the decimal system. It uses only ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 called digits. Every whole number can be written with the help of only these symbols.

CHAPTER THREE

Operations on Whole Numbers

WE ARE ALREADY FAMILIAR with the operations of addition, subtraction, multiplication and division on whole numbers. Let us now study some properties of these operations. These properties are simple, but useful for further studies in arithmetic and algebra

3.1 Properties of Addition

Property I: Let us add any two whole numbers using the symbol '+' and see whether the sum is a whole number.

Whole number	+	*Whole number*	=	*Sum*	*Whether whole number or not*
8	+	7	=	15	Yes
6	+	6	=	12	Yes
0	+	15	=	15	Yes

We conclude that if we add two whole numbers, we get the sum, also a whole number. In other words,

If a and b are two whole numbers and $a + b = c$, *then c must be a whole number.*

Property II: Let us now consider some more pairs of whole numbers, add the two whole numbers of each pair in the two different orders and see whether the sum remains the same. We take the number pairs 4, 8; 10, 12 and 6, 15 and we have

$4 + 8 = 12$ and also $8 + 4 = 12$

Thus, $4 + 8 = 8 + 4$

Again, 10 + 12 = 22 and 12 + 10 = 22

Thus, 10 + 12 = 12 + 10

Further, 6 + 15 = 21 and 15 + 6 = 21

Thus, 6 + 15 = 15 + 6

We conclude that whatever be the order, we may add two whole numbers, the sum remains the same. The table of addition facts also illustrates the same. We can thus have the table, called table of addition.

TABLE 3.1

Second Number

First Number

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5
2	2	3	4	5	6
3	3	4	5	6	7
4	4	5	6	7	8

Again, from the table we observe that

2 + 1 = 1 + 2, 3 + 2 = 2 + 3, 4 + 1 = 1 + 4, etc.

Can you say from the table why 2 + 1 and 1 + 2 are the same? What can we say about 3 + 2 and 2 + 3? And so on.

In general, we can thus state: *If a and b are two whole numbers, then a + b = b + a.*

In other words, the sum of two whole numbers remains the same even if the order of numbers (called addends) is changed.

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Property III: Let us once again observe the entries of addition facts given in Table 3.1. From the table, we note that

$0 + 0 = 0$

$0 + 1 = 1 + 0 = 1$

$0 + 2 = 2 + 0 = 2$

$0 + 3 = 3 + 0 = 3$, and so on.

We see that the sum of zero and any whole number is the number itself. In other words, we conclude:

If a is any whole number, then $a + 0 = 0 + a = a$.

This is called the Addition Property of zero and 0 is called the *identity element* for addition.

Property IV: Let us now consider the addition of three whole numbers. We know that to find the sum of three whole numbers, we first find the sum of two of them and then add the third to the sum. Thus, to find the sum of 2, 3 and 5, we proceed as follows:

$2 + 3 + 5 = (2 + 3) + 5 = 5 + 5 = 10$

What happens if we add 2 to the sum of 3 and 5. This means

$$2 + 3 + 5 = 2 + (3 + 5) = 2 + 8 = 10$$

We get the same sum, i.e. 10 in each case. Let us verify this fact by other whole numbers, say 2, 4, 8 and 5, 11, 13. We get

$(2 + 4) + 8 = 6 + 8 = 14$

$2 + (4 + 8) = 2 + 12 = 14$

Thus, $(2 + 4) + 8 = 2 + (4 + 8)$

Again, $(5 + 11) + 13 = 16 + 13 = 29$

$5 + (11 + 13) = 5 + 24 = 29$

Thus, $(5 + 11) + 13 = 5 + (11 + 13)$

In general, *if a, b, c are any three whole numbers, then*

$$(a + b) + c = a + (b + c).$$

Remark: We may write $a + b + c = a + (c + b)$;

Therefore, $(a + c) + b = a + (c + b)$. We usually write a + b + c, the sum of these numbers a, b, c

Many a time, this property helps us in deciding which two numbers should be added first so that calculations become easier. Let us, for example, find the sum of 133,698 and 867. To find the sum, we first add 133 and 867. (Why?)

$$\begin{aligned} 133 + 698 + 867 &= (133 + 867) + 698 \\ &= 1000 + 698 \\ &= 1698 \end{aligned}$$

On the other hand, if we first add 133 and 698 or 698 and 867, calculations are more difficult.

We make use of Property II and Property IV mentioned above, possibly several times if necessary, to add four or more numbers. It is not necessary at this stage to point out the property used at each step. What is important is the consequence that to find the sum of several numbers, they need not be added in the order they are given. We arrange them suitably and find their sum. You might have seen that while adding a long list of figures, one would generally perform the addition first from upwards to downwards and then check the sum by performing the addition from downwards to upwards. Note that one uses a combination of the second and the fourth properties mentioned above.

Example 1: Find the sum of 234, 197 and 103.

Solution:

$$\begin{aligned} 234 + 197 + 103 &= (197 + 103) + 234 \\ &= 300 + 234 \\ &= 534 \end{aligned}$$

Example 2: Add together 14, 438, 486 and 162.

Solution:

$$\begin{aligned} 14 + 438 + 486 + 162 &= (486 + 14) + (438 + 162) \\ &= 500 + 600 \\ &= 1100 \end{aligned}$$

EXERCISE 3.1

1. Fill in the blanks to make each of the following a true statement:
   (i) 1005 + 283 = ——— + 1005
   (ii) 300507 + 0 = ———
   (iii) 12345 + (679 + 321) = (679 + ——) + 321
2. Is the sum of any two odd numbers an odd number?
3. Is the sum of any two even numbers an even number?
4. Add each of the following and check by reversing the order of the addends:
   (i) 5628 + 39784 (ii) 923584 + 178
   (iii) 15409 + 112 + 591 (iv) 2359 + 641 + 10000
5. Determine the sum by suitable rearrangement:
   (i) 637 + 908 + 363
   (ii) 2062 + 353 + 1438 + 547
6. A magic square is an array of numbers having the same number of rows and columns and the sum of numbers in each row, column or diagonal being the same. Fill in the blank cells of the following magic square:

22		6	13	20
	10	12	19	
9	11	18	25	
15	17	24	26	
16			7	14

3.2 Properties of Subtraction

We know how to subtract a whole number from another whole number. We may say addition and subtraction are in a way inverse of each other, as in addition, we combine objects together and in subtraction, we take away. We also know that in whole numbers we cannot subtract a larger whole number from a smaller one. We now discuss some properties of subtraction.

Property I: Let us study the following cases of subtraction of whole numbers:

Whole number	−	*Whole number*	=	*Difference*	*Whether whole number or not*
7	−	3	=	4	Yes
6	−	10	=	?	No
15	−	15	=	0	Yes
2	−	0	=	2	Yes
10	−	17	=	?	No

We find that if we subtract a whole number from another whole number, the difference may not always be a whole number. Thus, in general,

If a *and* b *are two whole numbers and* $a - b = c$ *also a whole number, then* $a > b$ *or* $a = b$*, otherwise subtraction is not possible in whole numbers.*

Property II: Let us now see whether the second property of addition is true for subtraction also. We have $4 - 3 = 1$ and $3 - 4 = ?$

Thus, $4 - 3$ is not equal to $3 - 4$.

$7 - 10 = ?$ $10 - 7 = 3$

Thus, $10 - 7$ is not equal to $7 - 10$.

In general, we conclude:

If a and b are two distinct whole numbers, then a − b is not equal to b − a.

Property III: We have 2 − 0 = 2, a whole number, but 0 − 2 = ?
6 − 0 = 6, is a whole number, but 0 − 6 = ?

We conclude that zero subtracted from any whole number gives the result as the number itself. However, any whole number (other then zero) subtracted from 0 is not the number itself. Thus, in general,

If a is any whole number, then a − 0 = a but 0 − a is not equal to a when a is not equal to zero.

We can now build up the table showing subtraction facts as follows:

TABLE 3.2

		Number to be subtracted									
Number from which to be subtracted	–	0	1	2	4	5	6	7	8	9	
	0	0									
	1	1	0								
	2	2	1	0							
	3	3	2	1	0						
	4	4	3	2	1	0					
	5	5	4	3	2	1	0				
	6	6	5	4	3	2	1	0			
	7	7	6	5	4	3	2	1	0		
	8	8	7	6	5	4	3	2	1	0	
	9	9	8	7	6	5	4	3	2	1	0

From Table 3.2 it is clear that 0 − 0 = 0, 1 − 0 = 1, 2 − 1 = 1 but 0 − 1, 1 − 3, 2 − 5, are not there.

Property IV: Let us take three whole numbers 7, 4 and 2. If we subtract 2 from 4 and again subtract this difference from 7, we get the result 5. If we subtract 4 from 7 and then subtract 2 from the difference, we get the result 1. We find that the final results are not the same.

Hence, Property IV for addition is not in general applicable for subtraction. Thus,

If a, b, c are three whole numbers, in general, $a-(b-c)$ is not equal to $(a-b)-c$.

EXERCISE 3.2

1. Perform the following subtractions, check your results by corresponding additions:
   (i) 7839 − 983　　(ii) 12304 − 10999
   (iii) 100000 − 98765　　(iv) 2020201 − 565656
2. Replace each * by the correct digit in each of the following:

(i)
$$\begin{array}{r} 876 \\ -\ {*}\,3\,{*} \\ \hline 6{*}7 \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 5376 \\ -\ {*}{*}59 \\ \hline 25{*}{*} \\ \hline \end{array}$$

(iii)
$$\begin{array}{r} 6000107 \\ -\ {*}{*}8978 \\ \hline 506{*}{*}{*}{*} \\ \hline \end{array}$$

(iv)
$$\begin{array}{r} 1000000 \\ -\ {*}{*}{*}{*}1 \\ \hline {*}4320{*} \\ \hline \end{array}$$

3. What is the difference between the largest number of five digits and the smallest number of six digits?
4. Saleem deposited Rs 25,000 in his savings bank account. Later he withdrew Rs 5,425 from it. How much money was left in his account?

5. The total number of men, women and children in a town is 96,209. If the number of men is 29,642 and that of women is 29,167, determine the number of children.

3.3 Properties of Multiplication

Property I: As in the case of addition we take a few pairs of whole numbers, and see whether the product is also a whole number.

Whole number	×	*Whole number*	=	*Product*	*Whether whole number or not*
7	×	8	=	56	Yes
0	×	1	=	0	Yes
11	×	11	=	121	Yes

We see that if we multiply two whole numbers, we get the product also a whole number. In other words, we conclude:

If a and b are two whole numbers and $a \times b = c$, *then c must be a whole number.*

Property II: We now consider some pairs of whole numbers, multiply the two whole numbers of each pair in two different orders and see whether the product remains the same. Let us suppose the number pairs are 2, 7; 0, 5 and 12, 20. We have

$2 \times 7 = 14$ and $7 \times 2 = 14$

So, $2 \times 7 = 7 \times 2$

$0 \times 5 = 0$ and $5 \times 0 = 0$

So, $0 \times 5 = 5 \times 0$

$12 \times 20 = 240$ and $20 \times 12 = 240$

So, $12 \times 20 = 20 \times 12$

We see that in whatever order we may multiply two whole numbers, the product remains the same. The table of multiplication facts also illustrates the same. As in the case of addition, let us consider only the first five whole numbers. We have the following table:

TABLE 3.3

Second Number

	$\times$	0	1	2	3	4
First Number	0	0	0	0	0	0
	1	0	1	2	3	4
	2	0	2	4	6	8
	3	0	3	6	9	12
	4	0	4	8	12	16

It readily follows that $2 \times 3 = 3 \times 2$, $3 \times 4 = 4 \times 3$, $1 \times 4 = 4 \times 1$, etc. The property illustrated above may be restated in general, as follows:

If a and b are two whole numbers, then

$$a \times b = b \times a.$$

In other words, the product of two numbers remains the same even if their order is changed.

Property III: From the table of multiplication facts given above, we observe that

$0 \times 0 = 0$
$0 \times 1 = 1 \times 0 = 0$
$0 \times 2 = 2 \times 0 = 0$, and so on.

We see that the product of any whole number and zero is zero. In other words,

If a is any whole number, then $a \times 0 = 0 \times a = 0$.

Property IV: From Table 3.3 we also observe that

$1 \times 0 = 0 \times 1 = 0$
$1 \times 1 = 1$
$1 \times 2 = 2 \times 1 = 2$
$1 \times 3 = 3 \times 1 = 3$, and so on.

We conclude that the product of any whole number and 1 is the number itself. In other words,

If a is any whole number, then $1 \times a = a \times 1 = a$.

This is the *multiplication property* of 1 and 1 is called the *identity element for multiplication.*

Property V: We now concentrate on multiplying three whole numbers and see whether multiplying them in different orders makes any difference or not. We know that to find the product of three numbers, we first multiply two of them and then multiply the product obtained by the third number. Let us take the numbers 3, 4 and 7. We We have

$$\begin{aligned} 3 \times 4 \times 7 &= (3 \times 4) \times 7 \\ &= 12 \times 7 \\ &= 84 \end{aligned}$$

We now change the arrangement and have

$$\begin{aligned} 3 \times 4 \times 7 &= 3 \times (4 \times 7) \\ &= 3 \times 28 \\ &= 84 \end{aligned}$$

We get the same product, i.e. 84 in each case. Let us verify this fact by other sets of whole numbers, say, 2, 5, 9 and 3, 10, 12. We get

$$(2 \times 5) \times 9 = 10 \times 9 = 90$$

$$2 \times (5 \times 9) = 2 \times 45 = 90$$

Thus, $(2 \times 5) \times 9 = 2 \times (5 \times 9)$

Also,

$$(3 \times 10) \times 12 = 30 \times 12 = 360$$

$$3 \times (10 \times 12) = 3 \times 120 = 360$$

Thus, $(3 \times 10) \times 12 = 3 \times (10 \times 12)$

In view of the above, we conclude that to multiply three whole numbers, we may change the arrangements of the numbers, but the product remains the same. In other words, in general, we conclude:

If a, b, c are any three whole numbers, then

$$(a \times b \times c = a(b \times c)$$

We *usually write* $a \times b \times c$ *for these equal products.*

Many times, this property helps us in deciding which two numbers should be multiplied first so that calculations are easier. Let us, for example, find the product of 8, 1769 and 125. To find their product, we first multiply 8 and 125. (Why?)

$$\begin{aligned} 8 \times 1769 \times 125 &= (8 \times 125) \times 1769 \\ &= 1000 \times 1769 \\ &= 1769000 \end{aligned}$$

On the other hand, if we first multiply 8 and 1769 or 125 and 1769, calculations are more difficult.

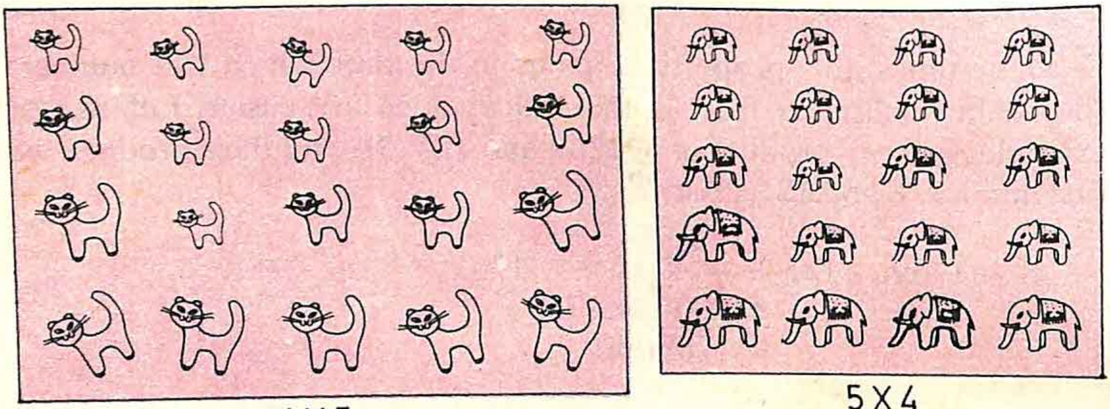
We make use of the second and the fifth properties of multiplication mentioned above, possibly several times, if necessary, to multiply four or more numbers. It is not important at this stage to point out the property used at each step. What is important is the consequence that to *find the product of several numbers, they need not be multiplied in the order they are given. We group them in easy combinations and find their product.* This we may call the *rearrangment property* of multiplication.

Example 3: Find the product: $15237 \times 25 \times 80 \times 16$.

Solution:

$$\begin{aligned} 15237 \times 25 \times 80 \times 16 &= (25 \times 80) \times (16 \times 15237) \\ &= 2000 \times 243792 \\ &= 2000 \times 243792 \\ &= 487584000 \end{aligned}$$

Property VI: Finally, we derive a very important property which helps us in discovering the process of multiplying numbers containing two or more digits. We know how to represent a multiplication fact pictorially (see Fig. 3.1).

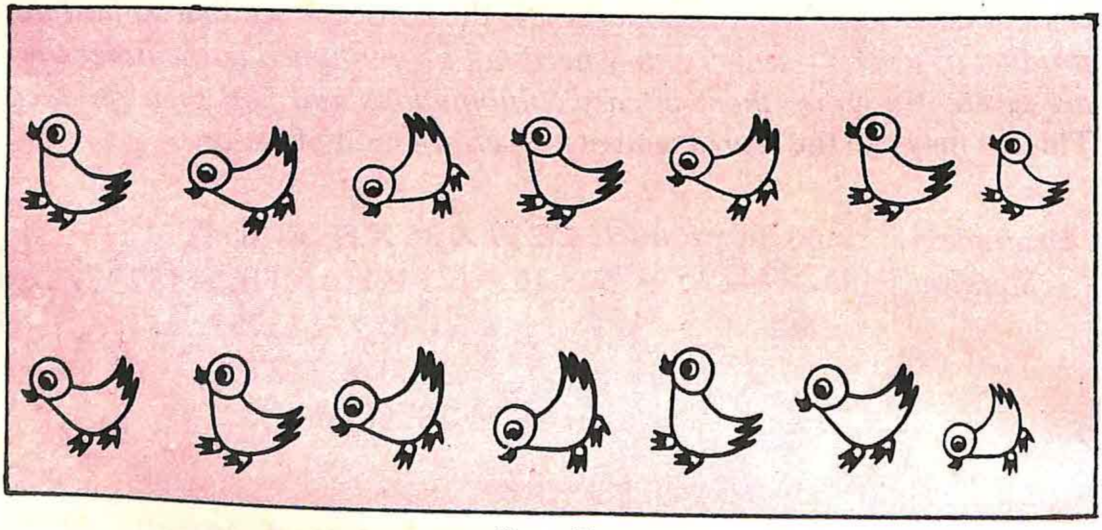


4X5

5X4

Fig. 3.1

Let us now consider the figure given below. It contains 2 rows of 7 birds drawn on a piece of paper and thus represents the multiplication fact 2 × 7.



2 × 7

Fig. 3.2

Let us fold our paper so that we separate 7 birds in each row to show that 7 birds are the same as 3 birds and 4 birds (see Fig. 3.3). The crease separates the multiplication fact 2 × 7 into two multiplication facts 2 × 3 and 2 × 4.

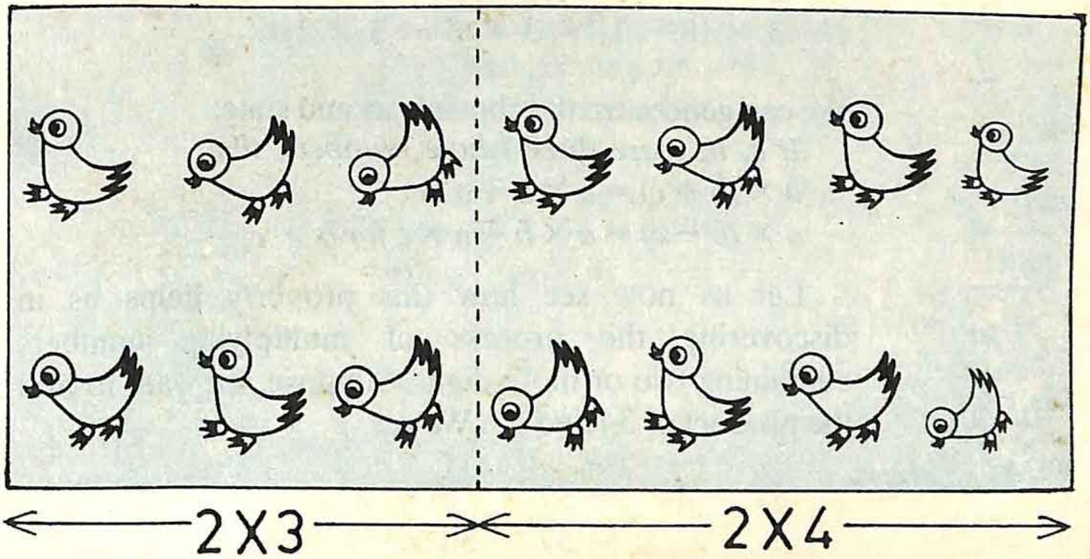


Fig. 3.3

Figs. 3.2 and 3.3 help us to see

$2 \times 7 = 2 \times 3 + 2 \times 4$

Also, $2 \times 7 = 2 \times (3 + 4)$

Therefore, $2 \times (3 + 4) = 2 \times 3 + 2 \times 4$

Let us verify this fact my means of other examples. We have

$3 \times (5+7) = 3 \times 12 = 36$

$3 \times 5 + 3 \times 7 = 15 + 21 = 36$

Therefore, $3 \times (5+7) = 3 \times 5 + 3 \times 7$

Also, $4 \times (8 + 9) = 4 \times 17 = 68$

$4 \times 8 + 4 \times 9 = 32 + 36 = 68$

Therefore, $4 \times (8+9) = 4 \times 8 + 4 \times 9$

The above-mentioned fact is *called distributive property of multiplication over addition.* One can easily verify that this property is true over subtraction also. For example,

$$4 \times (5 - 3) = 4 \times 5 - 4 \times 3$$
$$2 \times (9 - 4) = 2 \times 9 - 2 \times 4$$
$$3 \times (8 - 1) = 3 \times 8 - 3 \times 1$$

We can generalize the above facts and state:

If a, b, c are three whole numbers, then

$$a \times (b + c) = a \times b + a \times c$$
$$a \times (b - c) = a \times b - a \times c \text{ for } b > c$$

Let us now see how this property helps us in discovering the process of multiplying numbers containing two or more digits. Suppose, we want to find the product of 34 and 57. We get

$$\begin{aligned} 57 \times 34 &= 57 \times (30 + 4) \\ &= 57 \times 30 + 57 \times 4 \\ &= 1710 + 228 \\ &= 1938 \end{aligned}$$

Therefore, to multiply 57 by 34, we first find the product 57 × 4 and then 57 × 30. The sum of these products gives us the required product of multiplying 57 by 34. Let us give the method in various steps as follows:

Step 1: We multiply 57 by 4 (one's digit of the multiplier) and write the product (i.e. 228) as on the right

$$\begin{array}{r} 57 \\ \times\ 34 \\ \hline 228 \end{array}$$

Step 2: Next we multiply 57 by 30 (3 tens), i.e. we multiply 57 by 3 (ten's digit of the multiplier) and insert a zero on the right of the product of 57 and 3. We, thus, write the product 57×30, i.e. 1710 directly under the product obtained in Step 1.

$$\begin{array}{r} 57 \\ \times\ 34 \\ \hline 228 \\ 1710 \end{array}$$

Step 3: Finally, we add the two products (here 228 and 1710) and get the sum 1938 as the required product of 57 and 34.

$$\begin{array}{r} 57 \\ \times\ 34 \\ \hline 228 \\ 1710 \\ \hline 1938 \\ \hline \end{array}$$

Thus, $57 \times 34 = 1938$

Example 4: Multiply 375 by 84.

Solution:

$$\begin{array}{rl} 375 & \\ \times\ 84 & \\ \hline 1500 & \leftarrow \text{This is the product of 375 and 4.} \\ 30000 & \leftarrow \text{This is the product of 375 and 80.} \\ \hline 31500 & \leftarrow \text{This is the sum of 1500 and 30000.} \end{array}$$

Therefore, $375 \times 84 = 31500$

Sometimes the distributive property helps us in simplifying some expressions. Let us study the following examples.

Example 5: Find the value of $467 \times 97 + 467 \times 3$

Solution:

$$\begin{aligned} 467 \times 97 + 467 \times 3 &= 467 \times (97 + 3) \\ &= 467 \times 100 \\ &= 46700 \end{aligned}$$

Example 6: Find the product 615×96.

Solution: $615 \times 96 = 615 \times (100 - 4)$

$= 615 \times 100 - 615 \times 4$

$= 61500 - 2460$

$= 59040$

Remark: The distributive property of multiplication over addition holds good in case the number of addends inside the parentheses is more than two. For example,

$2 \times (3 + 4 + 5) = 2 \times 3 + 2 \times 4 + 2 \times 5$.

Example 7: Find the value of

$4938 \times 887 + 108 \times 4938 + 4938 \times 5$

Solution: The given expression $= 4938 \times (887 + 108 + 5)$

$= 4938 \times 1000$

$= 4938000$

Example 8: Simplify: $25 + 30 - 7 \times (15 \div 3) + 5$

Solution: We know that in simplifying numerical expressions containing various operations, division and multiplication are performed first, followed by addition and subtraction. Thus, the given expression

$= 25 + 30 - 7 \times 5 + 5$

$= 25 + 30 - 35 + 5$

$= 60 - 35 = 25$

EXERCISE 3.3

1. Fill in the blanks to make each of the following a true statement:

(i) $15379 \times 0 =$ ———

(ii) $675 \times 47 = 47 \times$ ———

(iii) $3709 \times 1 =$ ———

(iv) $10 \times 100 \times$ ——— $= 10000$

(v) $42 \times 18 \times 15 = 18 \times$ ——— $\times 42$

(vi) $45 \times 36 = 45 \times 30 + 45 \times$ ———

(vii) $27 \times 18 = 27 \times 9 + 27 \times$ ——— $+ 27 \times 5$

(viii) $12 \times 45 = 12 \times 50 - 12 \times$ ———

(ix) $66 \times 85 = 66 \times 90 - 66 \times$ ——— $- 66$

2. Determine the product by suitable rearrangement:
   - (i) $2 \times 1735 \times 50$ (ii) 166×425
   - (iii) $8 \times 291 \times 125$ (iv) $279 \times 625 \times 16$
   - (v) $285 \times 5 \times 60$ (vi) $125 \times 40 \times 8 \times 25$
3. Choose any two odd numbers. Is their product an odd number? Is it true for any two odd numbers?
4. Is the product of two even numbers always an even number?
5. We know that $0 + 0 = 0$. Is there some other whole number p such that $p + p = p$?
6. We know that $0 \times 0 = 0$. Is there some other whole number q such that $q \times q = q$.
7. ·Given that the product of two whole numbers is zero, what conclusion can you draw?
8. Find each of the following products:
   - (i) 816×745 (ii) 2032×613 (iii) 49381×206
   - (iv) 23701×4389
9. Using distributive property, find each of the following products:
   - (i) 736×103 (ii) 854×96 (iii) 258×1008
   - (iv) 995×158
10. Find the value of each of the following using properties:
    - (i) $297 \times 7 + 297 \times 3$
    - (ii) $54279 \times 92 + 8 \times 54279$
    - (iii) $8165 \times 169 - 8165 \times 69$
    - (iv) $15625 \times 15625 - 15625 \times 5625$
    - (v) $3845 \times 5 \times 782 + 769 \times 25 \times 218$
    - (vi) $887 \times 10 \times 461 - 361 \times 8870$
    - (vii) $579 \times 7 + 579 \times 2 + 579$
    - (viii) $16 \times 739 \times 6 - 12 \times 739$
    - (ix) $461 \times 999 + 461$
    - (x) $36 \times 583 \times 17 - 48 \times 583 - 5 \times 583$
11. Determine the product of the greatest number of four digits and the smallest number of three digits.
12. A dealer purchased 125 colour television sets. If the cost of each set is Rs 9820, determine the cost of all the sets together.

13. The annual fee charged from a student of Class VI in a school is Rs 80. If there are, in all, 235 students in Class VI, find the total collection.

3.4 Properties of Division

Property I: We already know how to divide a whole number by another smaller non-zero whole number. Let us see whether on dividing a whole number by another non-zero whole number we always get the result (quotient) a whole number or not.

We have $8 \div 2 = 4$

$7 \div 3 = 2$, remainder 1

$3 \div 5 = ?$

We notice that $7 \div 3$ and $3 \div 5$ do not represent whole numbers, therefore, we conclude:

If a and b (b not equal to zero) are whole numbers, then a ÷ b (expressed as $\frac{a}{b}$*) is not necessarily a whole number.*

Property II: We find that 'division' is in a way a repeated subtraction. For example, 3 can be repeatedly subtracted from 12 four times as shown below on the right and we write this fact as

$12 \div 3 = 4$

$$\begin{array}{rl} 12 & \\ -\ 3 & \text{one time} \\ \hline 9 & \\ -\ 3 & \text{two times} \\ \hline 6 & \\ -\ 3 & \text{three times} \\ \hline 3 & \\ -\ 3 & \text{four times} \\ \hline 0 & \\ \hline \end{array}$$

Let us try to subtract 0 repeatedly from a given whole number, say 6, and move towards 0.

$$\begin{array}{rl} 6 & \\ -\ 0 & \text{one time} \\ \hline 6 & \\ -\ 0 & \text{two times} \\ \hline 6 & \\ -\ 0 & \text{three times} \\ \hline 6 & \end{array}$$

We see that even if we subtract 0 from 6 any number of times, we remain at 6, i.e. we are unable to move towards zero. We, therefore, say:

Division by zero is not defined.

Property III: We also find that division is inverse of multiplication. For example,

$$6 \div 3 = 2 \quad \text{since} \quad 2 \times 3 = 6$$
$$24 \div 8 = 3 \quad \text{since} \quad 3 \times 8 = 24$$

For every multiplication fact for two distinct non-zero numbers there are two corresponding division facts.

Multiplication fact	*Corresponding division facts*
$3 \times 5 = 15$	$15 \div 3 = 15, 15 \div 5 = 3$
$4 \times 7 = 28$	$28 \div 4 = 7, 28 \div 7 = 4$

Since $1 \times 2 = 2$ we get $2 \div 1 = 2$, since $1 \times 3 = 3$ we get $3 \div 1 = 3$, and so on. We conclude:

Any whole number divided by 1 gives the quotient as the number itself.

In other words,

If a is any whole number $a \div 1 = a$.

Property IV: Let us again use the facts mentioned above and derive one more property. We have

$3 \div 3 = 1$ since $3 = 3 \times 1$

$4 \div 4 = 1$ since $4 = 4 \times 1$

and so on. We conclude:

Any whole number (other than zero) divided by itself gives the quotient 1.

In other words,

If a is a whole number (other than zero), then

$$a \div a = 1.$$

Property V: We have $0 \times 5 = 0$, therefore, we get $0 \div 5 = 0$. Similarly, $0 \div 6 = 0$, $0 \div 12 = 0$, etc. Thus,

Zero divided by any whole number (other than zero) gives the quotient zero.

In other words,

If a is a whole number (other than zero), then

$$0 \div a = 0.$$

Property VI: Using repeated subtraction let us divide 14 by 5. We get

$14 \div 5 = 2$, remainder 4

We note that $14 = 5 \times 2 + 4$

$$\begin{array}{rl} 14 & \\ -5 & \text{one time} \\ \hline 9 & \\ -5 & \text{two times} \\ \hline 4 & \end{array}$$

The number (here 14) which is to be divided is called *dividend.* The number (here 5) by which the dividend is divided is called *divisor.* The number of times (here 2) the divisor is contained in the dividend is called *quotient.* The number (here 4) which is left over after division is called *remainder.* In the above case, we see that

Dividend = Divisor × Quotient + Remainder

By means of examples, this can be easily verified. If whole number a be the dividend, b the divisor, q the quotient and r the remainder, then *algorithm* or *rule for division* of a by non-zero b is

$$a = bq + r$$

$$\text{where} \quad r < b$$

Example 9: Divide 46087 by 356 and check the result by division algorithm.

Solution: We get

$$46087 \div 356 = 129, \text{ remainder } 163$$

$$\begin{array}{r@{\,}r} & 129 \\ \cline{2-2} 356 \big) & 46087 \\ & 356 \\ \cline{2-2} & 1048 \\ & 712 \\ \cline{2-2} & 3367 \\ & 3204 \\ \cline{2-2} & 163 \end{array}$$

Check: $356 \times 129 + 163$

$= 45924 + 163$

$= 46087$

EXERCISE 3.4

1. Does there exist a whole number n such that $n \div n = n$?
2. Given that a and b are two distinct non-zero whole numbers, is $a \div b = b \div a$?
3. Divide and check the quotient and remainder:
   (i) $7772 \div 58$ (ii) $6906 \div 35$
   (iii) $96324 \div 245$ (iv) $12345 \div 975$
   (v) $16025 \div 1000$ (vi) $92845 \div 300$
4. Find the value of
   (i) $32475 \div 1$
   (ii) $0 \div 719$
   (iii) $476 + 620 \div 62$

(iv) 694 − 725 ÷ 725
(v) 1465 ÷ 1465 − 1465 ÷ 1465
(vi) 72450 ÷ (583 − 58)
(vii) (15625 ÷ 125) ÷ 125
(viii) 1400 − 200 × (150 ÷ 50)

5. Which least number should be subtracted from 1000 so that 30 divides the difference exactly?
6. Which least number should be added to 1000 so that 45 divides the sum exactly?
7. Which least 6-digit number is exactly divisible by 75?
8. Which greatest 4-digit number is exactly divisible by 40?
9. Find a number which when divided by 35 gives the quotient 20 and the remainder 18.
10. A gardener plans to plant 570 trees in 19 rows each containing the same number of trees, how many trees will there be in each row?
11. The population of a town is 4,50,772. 1 out of every 14 is reported to be educated. In all, how many educated persons are there in the town?
12. A cinema hall is to be constructed in which each row must have 36 seats. Determine the minimum number of rows required to make 600 persons seated at one time.
13. The cost price of 24 radio sets is Rs 18,720. Determine the cost price of one radio set if each costs the same.
14. Which of the following statements are true?
    (i) Every multiplication fact gives two corresponding division facts.
    (ii) A whole number divided by another whole number greater than 1 never gives the quotient equal to the former.
    (iii) Any whole number divided by itself gives the quotient 1.
    (iv) There are no distinct whole numbers a, b, c such that a ÷ (b ÷ c) = (a ÷ b) ÷ c.

Things to Remember

If a, b, c, etc. are whole numbers, then

1. $(a + b)$ is a whole number.
2. $(a \times b)$ is a whole number.
3. $(a - b)$ may or may not be a whole number.
4. $a \div b$ may or may not be a whole number.
5. $a + b = b + a$
6. $a \times b = b \times a$
7. $a - b \neq b - a$ $(a \neq b)$ ($\neq$ stands for 'is not equal to'.)
8. $a \div b = b \div a$ $(a = b)$ $(a \neq 0, b \neq 0)$
9. In general, $a \div b \neq b \div a$
10. $a + (b + c) = (a + b) + c = (a + c) + b$
11. $a \times (b \times c) = (a \times b) \times c = (a \times c) \times b$
12. In general, $a - (b - c) \neq (a - b) - c$
13. In general, $a \div (b \div c) \neq (a \div b) \div c$
14. $a + 0 = 0 + a = a - 0 = a$
15. $a \times 0 = 0 \times a = 0$ and $0 \div a = 0$ $(a \neq 0)$
16. $a \times 1 = 1 \times a = a \div 1 = a$
17. $a \times (b + c) = a \times b + a \times c$ and $a \times (b - c) = a \times b - a \times c$ for $b > c$
18. If a is dividend, b $(b \neq 0)$ divisor, q quotient and r remainder, then $a = bq + r$

CHAPTER FOUR

Factors and Multiples

WE ARE ALREADY FAMILIAR with the concepts of factor, multiple, prime and composite numbers. In this chapter, we shall review basic ideas and properties related to them and extend our study to include other new properties.

4.1 Factors and Multiples

We recall that a *factor* of a number is an exact divisor of that number. The number is said to be a *multiple* of any of its factors. The factors of 6 are 1, 2, 3 and 6. 6 is a multiple of each of 1, 2, 3 and 6.

We speak of factors and multiples only in the case of natural numbers. We already know that multiplication facts of a number give its factors. For example, let us consider all multiplication facts of 12. We have

$$\begin{aligned} 12 &= 1 \times 12 \\ &= 2 \times 6 \\ &= 3 \times 4 \end{aligned}$$

Therefore, 1, 2, 3, 4, 6 and 12 are factors of 12. If we divide 12 by any one of 1, 2, 3, 4, 6, 12 we get the remainder 0. In view of this we conclude:

(i) *If a number 'b' is a factor of a number 'a' then 'a' divided by 'b' gives the remainder 0.*
(ii) *1 is a factor of every number.*
(iii) *Any number is a factor of itself.*
(iv) *If 'a' is a factor of 'b' and 'b' is a factor of 'a', then $a = b$.*

All multiples of 2 are called *even* numbers and those which are not multiples of 2 are called *odd* numbers. Thus, we have

even numbers : 2, 4, 6, 8, 10, 12, . . .
odd numbers : 1, 3, 5, 7, 9, 11, . . .

4.2 Prime and Composite Numbers

We classify the natural numbers into the following three categories:

(i) Numbers having exactly one factor
(ii) Numbers having exactly two distinct factors
(iii) Numbers having more than two factors.

The natural number 1 is the only number which has exactly one factor, the number 1 itself.

The numbers 2, 3, 5, 7, 11, 13, etc. come in the second category. Each of these numbers has exactly two distinct factors—the number 1 and the number itself. These are called *prime numbers.*

The numbers 4, 6, 8, 9, 10, 12, 14, 15, etc. come in the third category. Each of them has more than two factors. For example, 9 has three factors 1, 3, 9. These are called *composite numbers.*

Since the number 1 does not come in either of the last two categories, we say that *the number 1 is neither prime nor composite.*

We note that 2 is the lowest prime number and the only even prime number. All other even numbers are composite numbers.

A simple method for finding the prime numbers was found by *Eratosthenes,* a Greek mathematician in the third century B.C. The

~~1~~	②	③	~~4~~	⑤	~~6~~	⑦	~~8~~	~~9~~	~~10~~
⑪	~~12~~	⑬	~~14~~	~~15~~	~~16~~	⑰	~~18~~	⑲	~~20~~
~~21~~	~~22~~	㉓	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	㉙	~~30~~
㉛	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	㊲	~~38~~	~~39~~	~~40~~
㊶	~~42~~	㊸	~~44~~	~~45~~	~~46~~	㊼	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	(53)	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	(59)	~~60~~
(61)	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	(67)	~~68~~	~~69~~	~~70~~
(71)	~~72~~	(73)	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	(79)	~~80~~
~~81~~	~~82~~	(83)	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	(89)	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	(97)	~~98~~	~~99~~	~~100~~

method is known as the Sieve of Eratosthenes. We illustrate this for natural numbers 1 to 100. First we prepare a table of natural numbers 1 to 100 as given on page 45.

We cross out 1 as we know that 1 is not a prime number.

We then encircle 2 and cross out every other multiple of 2, i.e. 4,6, 8, 10, ...

Next uncrossed number is 3. So we encircle 3 and cross out every other multiple of 3, i.e. 6, 9, 12, 15, ... If a number like 6, 12 is already crossed out, we need not cross it out again.

The next uncrossed number is 5. So we encircle it and cross out every other multiple of 5, i.e. 10, 15, 20, ...

We do this for all numbers till every number is either encircled or crossed out.

In the table, all encircled numbers are prime numbers. Leaving aside 1, the numbers that are crossed out are composite numbers.

In the above list of prime numbers between 1 and 100, we come across pairs of prime numbers which have only one composite number between them. For example, the prime numbers 3 and 5 have 4 as the only composite number between them. Such pairs of prime numbers (sometimes called *primes*) are called *twin primes*. The other pairs of twin primes between 1 and 100 are

5, 7; 11, 13; 17, 19; 29, 31; 43; 59, 61 and 71, 73.

In 1742, the famous mathematician Goldbach had a conjecture (guess) for which he could not supply a proof. His conjecture stated:

Every even number greater than 4 can be expressed as a sum of two odd prime numbers.

This can be easily verified by means of examples: $6 = 3 + 3$, $8 = 3 + 5$, $10 = 3 + 7$, $12 = 5 + 7$, $14 = 3 + 11$, and so on. So far no mathematician could supply a proof or contradict it by finding even one example. It still remains a conjecture and is known as *Goldbach's conjecture.*

Example 1: Determine whether 27 is a factor of 6561.

Solution: We divide 6561 by 27 and see whether the remainder is 0.

$$\begin{array}{r}
27)\,6561\,(243 \\
\underline{54} \\
116 \\
\underline{108} \\
81 \\
\underline{81} \\
\underline{0}
\end{array}$$

Since the remainder is 0, we conclude that 27 is a factor of 6561.

Example 2: Which of the following numbers are primes?

204, 83, 139

Solution: (i) $204 = 1 \times 204$

$= 2 \times 102$

$= 3 \times 68$

$= 4 \times 51$, and so on

We find that 1, 2, 3, 4, etc. are all factors of 204. In other words, it has more than two factors. Hence, 204 is not a prime number.

(ii) 83 has the only multiplication fact $83 = 1 \times 83$. It is, therefore, a prime number as it has exactly two distinct factors 1 and 83.

(iii) $139 = 1 \times 139$

No other multiplication fact is possible. Therefore, 139 is also a prime number.

EXERCISE 4.1

1. Write all factors of each of the following:

(i) 60 (ii) 64 (iii) 76 (iv) 125 (v) 144 (vi) 729

2. Write the first five multiples of
   (i) 16 (ii) 17 (iii) 19 (iv) 20 (v) 25 (vi) 40
3. Which of the following have 15 as their factor?
   (i) 15625 (ii) 123015 (iii) 151230
4. Which of the following are divisible by 21?
   (i) 21063 (ii) 20163 (iii) 21630
5. List all prime numbers between
   (i) 1 and 30 (ii) 78 and 158 (iii) 160 and 200
6. How many even numbers are prime?
7. Which of the following numbers are prime?
   (i) 23 (ii) 26 (iii) 31 (iv) 51 (v) 109 (vi) 1729*
8. Can a composite number be odd? If yes, write the smallest odd composite number.
9. Write six consecutive composite numbers less than 100 so that there is no prime number between them.
10. The digit in the unit's place of a number is 5. If the number lies between 150 and 200 will it be composite or prime?
11. For a number, greater than 10, to be prime what may be the possible digits in the unit's place?
12. (i) Is there a number which has no factors?
    (ii) From 1 to 100, how many numbers have exactly three factors?
13. If the sum of all the factors of a number is two times the number, it is called a *perfect number*. The smallest perfect number is 6. There is only one more perfect number between 1 and 100. Determine it.
14. Determine pairs of twin primes, if any, between 100 and 150.
15. Express each of the following numbers as a sum of two odd primes:
    (i) 32 (ii) 40 (iii) 56 (iv) 80 (v) 96 (vi) 100

---

* The number 1729 is called Ramanujan's number. It is the smallest number which can be expressed as a sum of two cubes in two different ways—$12^3 + 1^3$ and $10^3 + 9^3$.

4.3 Tests for Divisibility of Numbers

To find whether or not a number is divisible by another, we usually perform the actual division and see whether or not the remainder is zero. This is both time-consuming and unnecessary. Much easier methods are available to test whether or not a number is divisible by certain other numbers. In earlier classes we have already learnt such divisibility tests of numbers by 10, 2, 3, 5 or 9. Simply by examining the digits in the number we were able to say definitely whether the number is divisible by any of these numbers. Let us review these tests here in brief.

(i) *Divisibility by 10*

A number is divisible by 10, if its unit's digit is zero. For example, each of the numbers 20, 130, 1500 is divisible by 10.

(ii) *Divisibility by 2*

A number is divisible by 2 if its unit's digit is 0, 2, 4, 6, or 8. For example each of the numbers 21674, 31856, 20018, 43560 is divisible by 2. We note that all even numbers are divisible by 2.

(iii) *Divisibility by 3*

A number is divisible by 3 if the sum of its digits is a multiple of 3. 5412 is divisible by 3 since 5 + 4 + 1 + 2, i.e. 12 is a multiple of 3. 343 is not divisible by 3 since 3 + 4 + 3, i.e. 10 is not a multiple of 3.

(iv) *Divisibility by 5*

A number is divisible by 5 if its unit's digit is either 0 or 5. For example, each of the numbers 155, 5370, 69135 is divisible by 5. On the other hand, if the unit's digit is other than 0 or 5, the number will not be divisible by 5. For example, none of the numbers 113, 2003, 5476 is divisible by 5.

(v) *Divisibility by 9*

A number is divisible by 9 if the sum of its digit is a multiple *of 9.* If it is not so, then the number will not be divisible by 9. The number 7281 is divisible by 9 since 7 + 2 + 8 + 1, i.e. 18 is a multiple of 9. The number 3478 is not divisible by 9 since 3 + 4 + 7 + 8, i.e. 22 is not a multiple of 9.

We now discuss some other properties of divisibility which help us in determining the divisibility of a number by some other numbers.

4.4 Some General Properties of Divisibility

Property I: We know that 72 is divisible by 12 since

$$72 = 12 \times 6$$

Also, $12 = 3 \times 4$

Therefore, 72 should be divisible by 3.

It is so, for $72 = 3 \times 24$

Similarly, 72 should be divisible by 4.

It is so, for $72 = 4 \times 18$

Let us consider one more example. We know that 40 is divisible by 10 since $40 = 10 \times 4$

Also, $10 = 2 \times 5$

Therefore, 40 must be divisible by both 2 and 5. It is so, for $40 = 2 \times 20$ and $40 = 5 \times 8$

We conclude:

If a number is divisible by another number, it must be divisible by each of the factors of that number.

The above-mentioned property, in other words, may be stated as follows:

If a, b, c are three natural numbers such that a is divisible by b and b is divisible by c, then a must be divisible by c.

Property II: We recall that two numbers are said to be *coprime* if they do not have a common factor other than 1. For example, the numbers 8 and 15 are coprime as their only common factor is 1.

Factors of 8 : 1, 2, 4, 8
Factors of 15 : 1, 3, 5, 15

It seems quite reasonable that if each of two or more coprime numbers exactly divides a given number, then their product would also exactly divide that number. Let us verify:

2 is a factor of 48 since 48 = 2 × 24
3 is a factor of 48 since 48 = 3 × 16
2 × 3, i.e. 6 is a factor of 48 since 48 = 6 × 8

We can verify this fact by means of other examples and thus arrive at the following important conclusion:

If a number is divisible by each of the two or more coprime numbers, it must be divisible by their product.

It is to be noted here that the divisors must be coprime numbers. If they are not coprime, the above-mentioned property may not be true. Let us, for example, consider the following situation:

4 is a factor of 12 since 12 = 4 × 3
6 is a factor of 12 since 12 = 6 × 2

But 4 × 6, i.e. 24 is not a factor of 12. It is so, because the numbers 4 and 6 are not coprime, and apart from 1, they have 2 as another common factor.

The above-mentioned property can be restated as follows:

If a and b are two coprime numbers, and both a and b are factors of c then a × b must be a factor of c.

This property is important because we can test the divisibility of a number by another number by testing its divisibility by such coprime factors of that number whose product is the number itself. For example, in order to test whether 38520 is divisible by 45, we test the divisibility of 38520 by 5 and 9 since 45 = 5 × 9 where 5 and 9 are coprime. Since 38520 is divisible by both 5 and 9, we conclude that it is divisible by 45.

Property III: Using distributive property of multiplication over addition we now derive one more important property of divisibility. Let us consider the following example. We have 5 as a factor of 20 as well as of 15.

$$20 = 5 \times 4$$
$$15 = 5 \times 3$$

Adding the two results, we get

$$\begin{aligned} 20 + 15 &= 5 \times 4 + 5 \times 3 \\ &= 5 \times (4 + 3) \\ &= 5 \times 7 \quad \text{(Why?)} \end{aligned}$$

We find that 5 being the common factor of 20 and 15 is also a factor of the sum of 20 and 15, i.e. 35, as well.
We conclude:

If a number is a factor of each of the two given numbers, it must be a factor of their sum.

In other words, this property can be stated as follows:

If 'a' is a factor of both 'b' and 'c' then 'a' must be a factor of 'b + c'.

Property IV: We know that distributive property of multiplication over subtraction also holds good. In the above example, subtracting 15 from 20, we get

$$\begin{aligned} 20 - 15 &= 5 \times 4 - 5 \times 3 \\ &= 5 \times (4 - 3) \\ &= 5 \times 1 \quad \text{(Why?)} \end{aligned}$$

We find that 5 is a factor of the difference of 20 and 15 also. We conclude:

If a number is a factor of each of the two given numbers, it must be a factor of their difference.

This property, in other words, can be stated as follows:

If 'a' is a factor of both 'b' and 'c', then 'a' must be a factor of 'b − c'.

With the help of the above-mentioned general properties of divisibility, we now establish the divisibility tests by some numbers other than those which we have already learnt.

4.5 Divisibility Tests by 4, 6, 8 and 11

(i) *Divisibility by 4*

We know that 100 is divisible by 4 since $100 \div 4 = 25$. Let us now consider the divisibility of 43752 by 4.

$$\begin{aligned} 43752 &= 43700 + 52 \\ &= 437 \times 100 + 52 \end{aligned}$$

437×100 is divisible by 4 since $\dfrac{437 \times 100}{4} = 437 \times 25$

Also, 52 is divisible by 4 since $52 \div 4 = 13$

Therefore, by the third general property mentioned above $437 \times 100 + 52$, i.e. 43752 is divisible by 4.

We note that every natural number can be written in the form 'a natural number × 100 + the number formed by the digits in ten's and unit's places'. Since a number × 100 is always divisible by 4 we conclude:

A number is divisible by 4 if the number formed by its digits in ten's and unit's places is divisible by 4.

Example 3: Is 579964 divisible by 4?

Solution: The number formed by ten's and unit's digits is 64. Since 64 is divisible by 4, 579964 is divisible by 4.

(ii) *Divisibility by 6*

We know that $6 = 2 \times 3$ and the numbers 2 and 3 are coprime. Therefore, by the second general property of divisibility mentioned above, we conclude:

A number is divisible by 6 if it is divisible by both 2 and 3.

(iii) *Divisibility by 8*

As in the case of divisibility by 4, here also we take one example. Let us consider the divisibility of 6437536 by 8. We know that 1000 is divisible by 8 since 1000 ÷ 8 = 125. So we rewrite the given number in the form given below:

$$\begin{aligned} 6437536 &= 6437000 + 536 \\ &= 6437 \times 1000 + 536 \end{aligned}$$

6437 × 1000 is divisible by 8. (Why?) Also, 536 is divisible by 8 since 536 ÷ 8 = 67. Hence 6437536 is divisible by 8.

The number 536 is formed by the hundred's, ten's and unit's digits of the given number. We, therefore, conclude:

A number is divisible by 8 if the number formed by its digits in hundred's, ten's and unit's places is divisible by 8.

Example 4: Which of the following numbers are divisible by 8?

(i) 569048 (ii) 72148

Solution: (i) The number formed by the hundred's, ten's and unit's digits of 569048 is 048, i.e. 48. Since 48 is divisible by 8, 569048 is divisible by 8.

(ii) We find that 148 is not divisible by 8. Hence, 72148 is not divisible by 8.

(iv) *Divisibility by 11*

Let us consider some multiples of 11. Each of 308, 1331 and 61809 is a multiple of 11. It appears that difference between the sums of the digits in odd places and in even places of these numbers is either a '0' or a multiple of 11.

Number	*Sum of the digits in odd places*	*Sum of the digits in even places*	*Difference of the two sums*
308	3 + 8	0	(3 + 8) − 0 = 11
1331	1 + 3	3 + 1	(1 + 3) − (3 + 1) = 0
61809	9 + 8 + 6	0 + 1	(9 + 8 + 6) − (0 + 1) = 22

How should we be sure of this fact? It can be easily verified that any of the numbers 99,9999,999999, etc. is divisible by 11 and any of the numbers 11, 1001, 100001, etc. is also divisible by 11. Let us now consider the divisibility of a number, say 586949, by 11. We express the number in expanded notation.

$$586949 = 5 \times 100000 + 8 \times 10000 + 6 \times 1000 + 9 \times 100 + 4 \times 10 + 9$$

We rewrite it as

$$\begin{aligned} 586949 &= 5 \times (100001 - 1) + 8 \times (9999 + 1) + 6 \times (100 - 1) + 9 \times (99 + 1) + 4 \times (11 - 1) + 9 \\ &= 5 \times 10001 + 8 \times 9999 + 6 \times 1001 + 9 \times 99 + 4 \times 11 + 9 - 4 + 9 - 6 + 8 - 5 \quad \text{(Why?)} \\ &= 5 \times 100001 + 8 \times 9999 + 6 \times 1001 + 9 \times 99 + 4 \times 11 + (9 + 9 + 8) - (4 + 6 + 5) \end{aligned}$$

We have expressed 586949 in such a way that each of the first five terms is divisible by 11. Therefore, using the third general property of divisibility, we can say that the number will be divisible by 11 if only $(9 + 9 + 8) - (4 + 6 + 5)$ is divisible by 11. Since $(9 + 9 + 8) - (4 + 6 + 5)$ is divisible by 11, we find that the number 586949 is divisible by 11. We note that 9, 9, 8 are the digits in odd places starting from unit's place and 4, 6, 5 those in even places. We conclude:

A number is divisible by 11 if the difference of the sum of its digits in odd places and the sum of its digits in even places (starting from unit's place) is either 0 or a multiple of 11.

Example 5: Test whether 8050314052 is divisible by 11?

Solution: Sum of the digits in even places

$= 5 + 4 + 3 + 5 + 8$

$= 25$

Sum of the digits in odd places

$= 2 + 0 + 1 + 0 + 0$

$= 3$

The difference of these sums $= 25 - 3 = 22$

Since 22 is divisible by 11, therefore, the given number is divisible by 11.

Note: It can be easily verified that a number is prime if none of the smaller prime numbers is a factor of it.

Example 6: Determine the largest prime number that you need to test as a divisor to find whether or not 401 is a prime number.

Solution: We have prime numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, . . .

401 is obviously not divisible by 2, 3, 5 (Why)?
401 is not divisible by 7 as we have
$401 \div 7 = 57$, remainder 2
401 is not divisible by 11 (Why?)
401 is not divisible by 13 since $401 \div 13 = 30$, remainder 11.
Similarly, 401 is not divisible by 17 and 19.
Also, $401 \div 19 = 21$, remainder 2.

The next prime number is 23 which is greater than the last quotient 21 obtained here. So we need not test its divisibility by 23.

Therefore, the required largest prime number is 19, and we find that 401 is a prime number.

EXERCISE 4.2

1. Using divisibility tests determine which of the following numbers are divisible by 2; by 3; by 5; by 9:
   (i) 126 (ii) 672 (iii) 990 (iv) 2050 (v) 2856 (vi) 406839
2. Using divisibility tests, determine which of the following numbers are divisible by 4; by 8:
   (i) 512 (ii) 12159 (iii) 4096 (iv) 14540
   (v) 21084 (vi) 31795012
3. Test the divisibility of the following numbers by 6:
   (i) 12583 (ii) 639210 (iii) 297144

4. Test the divisibility of the following numbers by 11:
   (i) 5335 (ii) 10824 (iii) 9020814 (iv) 3178965
   (v) 70169803 (vi) 10000001
5. Find the largest prime number that you need to test as a divisor to determine whether or not each of the following is a prime number:
   (i) 101 (ii) 203 (iii) 211

   In each case, also determine whether or not the number is a prime number.
6. Which of the following statements are true?
   - (i) If a number is divisible by 3, it must be divisible by 9.
   - (ii) If a number is divisible by 9, it must be divisible by 3.
   - (iii) All numbers divisible by 8 are also divisible by 4.
   - (iv) A number is divisible by 18 if it is divisible by both 3 and 6.
   - (v) If a number is divisible by both 9 and 10, it must be divisible by 90.
   - (vi) If a number exactly divides the sum of two numbers, it must exactly divide the numbers separately.
   - (vii) If a number divides three numbers exactly, it must divide their sum exactly.
   - (viii) If two numbers are coprime, at least one of them must be a prime number.
   - (ix) The sum of two consecutive odd numbers is always divisible by 4.

4.6 Prime Factorizations

We already know about factors and primes. Let us factorize 24 in three different ways as given below:

$$\begin{aligned} 24 &= 2 \times 12 \\ &= 2 \times 2 \times 6 \\ &= 2 \times 2 \times 3 \end{aligned} \qquad \begin{aligned} 24 &= 3 \times 8 \\ &= 3 \times 2 \times 4 \\ &= 3 \times 2 \times 2 \times 2 \end{aligned} \qquad \begin{aligned} 24 &= 4 \times 6 \\ &= 2 \times 2 \times 2 \times 3 \end{aligned}$$

In each case, 24 is expressed as a product of prime factors consisting of only 2 and 3.

Thus, each of the four factors of 24 is a prime number. We say that 24 has been expressed as a product of prime factors. Such factorizations are called *prime factorizations.* In other words,

A factorization is prime if all the factors are primes.

We observe that there are several prime factorizations of a number. We also notice that in each of the prime factorizations, the factors may be arranged differently, but the *prime factorization is unique.* We conclude:

Every composite number has exactly one prime factorization, irrespective of the order of its factors.

This property is called the *Prime Factorization Property* or the *Fundamental Theorem of Arithmetic.*

Example 7: Determine the prime factorization of 420.

Solution: We use division to extract prime factors of 420.

Unit's digit of 420 is 0. So it is divisible by 2.
We divide 420 by 2 and get 210 as quotient.
210 is also divisible by 2. (Why?)
So we divide it by 2 and get 105 as quotient.
Now, 105 is divisible by 3. (Why?)
So we divide it by 3 and get 35 as quotient.
The prime factors of 35 are obviously 5 and 7.
Thus, the required prime factorization is given by

$$\begin{array}{r|r} 2 & 420 \\ 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ 7 & 7 \\ \hline & 1 \end{array}$$

$$420 = 2 \times 2 \times 3 \times 5 \times 7$$

EXERCISE 4.3

1. Determine the prime factorization of each of the following numbers:

(i) 48 (ii) 34 (iii) 98 (iv) 216 (v) 525 (vi) 468
(vii) 441 (viii) 540 (ix) 9000 (x) 1024 (xi) 2145
(xii) 7325

2. Which factors are *not* included in the prime factorization of a composite number?
3. Write the smallest 5-digit number and express it as a product of primes.
4. Write the largest 4-digit number and give its prime factorization.
5. Find the prime factors of 1729. Arrange the factors in ascending order, and find the relation between two consecutive prime factors.

4.7 Highest Common Factor (H.C.F.)

The Highest Common Factor (H.C.F.) of two or more numbers is sometimes referred as their greatest common divisor (G.C.D.). We are already familiar with it. Let us find the H.C.F. of 12 and 16.

Factors of 12 : 1, 2, 3, 4, 6, 12
Factors of 16 : 1, 2, 4, 8, 16
Common factors of 12 and 16 : 1, 2, 4
Largest of the common factors, i.e. H.C.F. : 4

Thus, *the H.C.F. of two or more numbers is the largest or the highest among common factors.*

We recall that in earlier classes we found the H.C.F. of two or more numbers by prime factorization method. The method states:

We write prime factorization of each of the given numbers. We then find the product of all common prime factors of the numbers, using each common prime factor the least number of times it appears in the prime factorization of any of the numbers.

Example 8: Find the H.C.F. of 144, 180 and 192.

Solution: We write prime factorization of each of the given numbers:

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$180 = 2 \times 2 \times 3 \times 3 \times 5$

192 = 2 × 2 × 2 × 2 × 2 × 2 × 3

We note that 2 occurs as a prime factor at least two times in any of the given numbers and 3 at least once.

Therefore, the required H.C.F. = 2 × 2 × 3 = 12

It is obvious that prime factorization method of finding H.C.F. of certain numbers is convenient when the numbers are small. For larger numbers we use continued division method which we are going to discuss below by means of an example wherein we want to find the H.C.F. of 30 and 18.

Let us suppose we have two iron rods (AB and CD shown below) of lengths 30 cm and 18 cm, respectively, and we want to find the length of the largest rod by which we can measure each of these rods an exact number of times.

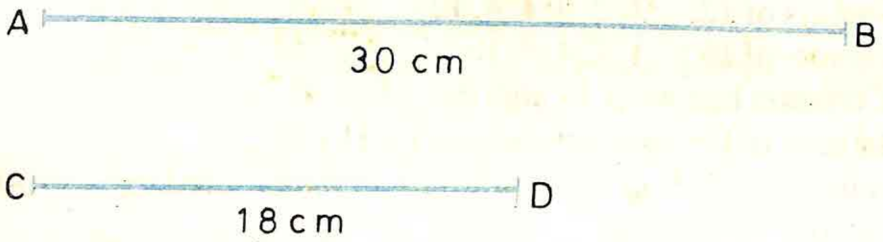


Fig. 4.1

Obviously, we can have a rod of length 1 cm and measure each of these rods an exact number of times. This gives rise to the fact that *when no prime factor is common to two given numbers, 1 will always be a common factor.* But we want the largest rod of that type. Let us see how we get the length of such a rod. Definitely, the length of such a rod must divide 30 cm and 18 cm exactly, i.e. it must be the H.C.F. of 30 cm and 18 cm.

We cut the larger rod into pieces, each equal to the length of the smaller rod. We get only one piece and a length of 12 cm is left. See the figure below:

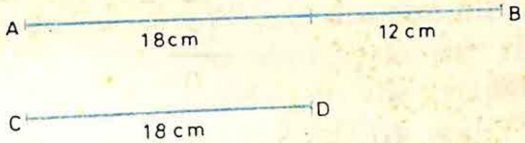


Fig. 4.2

We divide 30 by 18.

18) 30 (1
18
―――
12
―――

We get 1 as quotient and 12 as remainder

The rod which will measure CD an exact number of times, will obviously measure the 18 cm piece of AB an exact number of times.

Therefore, the required rod must measure the remaining portion of the larger rod, i.e. 12 cm and 18 cm an exact number of times.

Next, we cut the rod CD (18 cm) into pieces, each equal to the length of 12 cm. We get one such piece and a length of 6 cm is left. See the figure below:

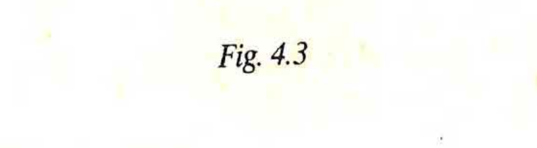


Fig. 4.3

Next we divide 18 (the first divisor) by 12 (the first remainder).

12) 18 (1
12
―――
6
―――

We get 1 as quotient and 6 as remainder.

By the same reasoning we get that the required rod must measure the remaining portion of CD, i.e. 6 cm and 12 cm an exact number of times.

We now cut the 12 cm piece into pieces, each equal to the length 6 cm. We get exactly two such pieces and no portion of the rod is left.

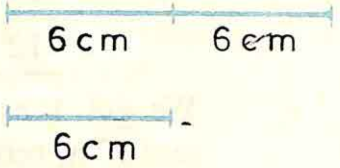


Fig. 4.4

We now divide 12 (the second divisor) by 6 (the second remainder).

```
6 ) 12 (2
    12
    ——
     0
    ——
```

We get 2 as quotient and 0 as remainder.

Obviously the required largest rod that will measure both the given rods an exact number of times must be of length 6 cm.

The required H.C.F. of 30 and 18 is 6 (the last divisor in the process when the remainder is zero).

We can summarize the above process as follows. To find the H.C.F. of 30 and 18 we divide 30 by 18, remainder is 12. Next we divide 18 (the divisor) by 12 (the remainder). The second remainder is 6. Then we divide 12 (the second divisor) by 6 (the second remainder) and get the remainder 0.

```
18) 30 (1
    18
    ——
    12) 18 (1
        12
        ——
         6 ) 12 (2
             12
             ——
              0
             ——
```

The last divisor, i.e. 6 (when the remainder is zero) is the required H.C.F. of 30 and 18.

Thus, we state the continued division method of finding the H.C.F. of two numbers as follows:

We divide the larger number by the smaller one getting a remainder. We then divide the first divisor by the remainder getting a new remainder. We again divide the next divisor by this remainder. This process goes on till at last we get the remainder zero. The last divisor in the process is the H.C.F. of the given numbers.

Example 9: Determine the H.C.F. of 216 and 1176.

Solution: We use division method as shown on the right and get the required H.C.F. of 216 and 1176 as 24.

```
216) 1176 (5
     1080
     ----
       96) 216 (2
           192
           ---
            24) 96 (4
                96
                --
                 0
                --
```

In order to find the H.C.F. of three numbers we find

(i) the H.C.F. of any two of them.

(ii) the H.C.F. of the third number and the H.C.F. obtained in (i) above.

The H.C.F. obtained in (ii) is the required H.C.F. of the three given numbers.

Example 10: Find the H.C.F. of 144, 180 and 192.

Solution:

```
144) 180 (1                    36 ) 192 (5
     144                             180
     ---                             ---
      36) 144 (4                      12) 36 (3
          144                             36
          ---                             --
            0                              0
          ---                             --
```

Hence the required H.C.F. of 144, 180 and 192 is 12.

Example 11: Find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively.

Solution: When 280 is divided by the number, a remainder of 4 is left. Therefore, 280/ − /4, i.e. 276 must be exactly divisible by the required number.

Similarly, 1245 − 3, i.e. 1242 must be exactly divisible by it.

Therefore, the required number must be the H.C.F. of 276 and 1242.

The H.C.F. of 276 and 1242 is 138.

Hence, the required number is 138.

```
276 )1242( 4
     1104
     ----
      138 ) 276 (2
            276
            ---
              0
            ---
```

Example 12: In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108, respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

Solution: The required number of persons in each room must be the H.C.F. of 60, 84 and 108.

The H.C.F. of 60, 84,108 is 12.

Therefore, in each room the highest number of persons to be seated is 12.

Hence, the number of rooms required

$$= \frac{60 + 84 + 108}{12}$$

$$= 21$$

EXERCISE 4.4

1. Determine the H.C.F. of numbers in each of the following by prime factorization method:
   - (i) 144, 198
   - (ii) 81, 117
   - (iii) 47, 61
   - (iv) 225, 450
   - (v) 13,39,273
   - (vi) 150, 140, 210
   - (vii) 120, 144, 204
   - (viii) 106, 159, 265
   - (ix) 101, 573, 1079
   - (x) 625, 3125, 15625
2. Determine the H.C.F. of numbers in each of the following by continued division method:
   - (i) 300, 450
   - (ii) 442, 1261
   - (iii) 252, 576
   - (iv) 935, 1320
   - (v) 1624, 522, 1276
   - (vi) 2241, 8217, 747
3. It is given that 65610 is divisible by 27. Which two numbers nearest to 65610 are each divisible by 27?
4. What is the H.C.F. of any two consecutive numbers?
5. Reduce each of the following fractions to the lowest terms by cancelling the H.C.F. of the numerator and the denominator:

   (i) $\frac{1444}{256}$ (ii) $\frac{143}{234}$ (iii) $\frac{2211}{5025}$
6. Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.
7. Two tankers contain 850 litres and 680 litres of petrol, respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times.
8. Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11 and 15, respectively.
9. The length, breadth and height of a room are 8m 25 cm, 6 m 75 cm and 4m 50cm, respectively. Determine the longest tape which can measure the three dimensions of the room exactly.
10. A rectangular courtyard is 20 m 16 cm long and 15 m 60 cm broad. It is to be paved with square stones of the same size. Find the least possible number of such stones.

4.8 Lowest Common Multiple (L.C.M.)

We recall that the lowest common multiple (L.C.M.) of two or more numbers is the smallest number which is a multiple of each of the given numbers. Let us find the L.C.M. of 8 and 12.

Multiples of 8 : 8, 16, 24, 32, 40, 48, . . .
Multiples of 12 : 12, 24, 36, 48, 60, 72, . . .

Common multiples of 8 and 12: 24, 48, . . .
The smallest of the common multiples of 8 and 12: 24

To find the L.C.M. of two or more numbers by prime factorization method, *we write prime factorization of each of the numbers. We then find the product of all the different prime factors of the numbers using each common prime factor the greatest number of times it appears in the prime factorization of any of the numbers.*

Example 13: Find the L.C.M. of 40, 36 and 126.

Solution: We write prime factorization of each number and have

$$40 = 2 \times 2 \times 2 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3$$
$$126 = 2 \times 3 \times 3 \times 7$$

We note that 2 occurs as a prime factor maximum three times, 3 two times, 5 one time and 7 one time. Therefore, required L.C.M. $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$$= 2520$$

We also use division method to find the L.C.M. of the given numbers. In this process, *we set aside all common factors of two or more given numbers by division. The product of all the common factors set aside and of those that remain is the required L.C.M. of the given numbers.*

Example 14: Determine the L.C.M. of 112, 140 and 168.

Solution: We use division method to find the L.C.M. of 112, 140 and 168 and have

2	112	140	168
2	56	70	84
2	28	35	42
7	14	35	21
	2	5	3

Therefore, L.C.M. $= 2 \times 2 \times 2 \times 7 \times 2 \times 5 \times 3$
$= 1680$

Example 15: Determine the lowest natural number which when divided by 16, 28, 40, 77 leaves remainder 8 in each case.

Solution: We know that the lowest number divisible by 16, 28, 40, 77 is their L.C.M. Therefore, the required number must be 8 more than their L.C.M. Now to find their L.C.M. we have

2	16	28	40	77
2	8	14	20	77
2	4	7	10	77
7	2	7	5	77
	2	1	5	11

Therefore, L.C.M. $= 2 \times 2 \times 2 \times 7 \times 2 \times 5 \times 11$
$= 6160$

Hence, the required number $= 6160 + 8$
$= 6168$

4.9 Some Properties of H.C.F. and L.C.M.

In view of our discussions in this chapter, we here list certain facts concerning the H.C.F. and the L.C.M. of numbers.

1. The H.C.F. of given numbers is not greater than any of the numbers.
2. The L.C.M. of given numbers is not less than any of the numbers.
3. H.C.F. of two coprime numbers is 1. Therefore, to determine whether or not two numbers are coprime we see whether or not their H.C.F. is 1.
4. Obviously, the L.C.M. of two or more coprime numbers is their product.
5. If a number, say a, is a factor of another number, say b, then the H.C.F. of a and b is a and their L.C.M. is b.
6. Since the H.C.F. of two or more numbers exactly divides each of the given numbers and each of the given numbers exactly divides their L.C.M., therefore, their H.C.F. must be a factor of their L.C.M. Consequently, there cannot exist two numbers whose H.C.F. is not a factor of their L.C.M.

We now discover a very important property concerning the H.C.F. and the L.C.M. of two numbers. Let us take the numbers 35 and 40.

H.C.F. of 35 and 40 : 5
L.C.M. of 35 and 40 : 280
Product of H.C.F. and L.C.M. $= 5 \times 280$
$= 1400$
Product of the given numbers $= 35 \times 70$
$= 1400$

Let us consider one more pair of numbers, say 50 and 60.

H.C.F. of 50 and 60 : 10
L.C.M. of 50 and 60 : 300
Product of H.C.F. and L.C.M. $= 3000$
Product of the numbers $= 50 \times 60$
$= 3000$

In each case we find that *the product of the H.C.F. and the L.C.M. of two numbers is equal to the product of the given numbers.* The proof of this property is beyond the scope of the present textbook. However, this can be verified by taking some other examples of numbers.

This property is sometimes used to find the L.C.M. of two very large numbers as we get

$$\text{L.C.M. of two numbers} = \frac{\text{Product of the numbers}}{\text{H.C.F. of the numbers}}$$

Example 16: On a particular day, from Delhi to Meerut buses ran at intervals of 40 minutes while from Meerut to Delhi they ran at intervals of 45 minutes. At what earliest time would two buses coming from opposite directions pass a particular bridge if they passed the same bridge at 10.15 a.m.?

Solution: The earliest time, in minutes, should be the L.C.M. of 40 and 45 added to 10.15 a.m.

L.C.M. of 40 and 45 = 360

Therefore, the two buses would have passed the same bridge after 360 minutes, i.e. 6 hours after 10.15 a.m. Hence, the two buses would pass the bridge at 4.15 p.m.

EXERCISE 4.5

1. Determine the L.C.M. of numbers in each of the following:
   (i) 48, 60 (ii) 18, 77
   (iii) 12, 15, 45 (iv) 15, 30, 90
   (v) 45, 105, 165 (vi) 6, 15, 18, 30
   (vii) 180, 384, 144 (viii) 240, 420, 660
   (ix) 108, 135, 162 (x) 112, 168, 266
2. For each of the following pairs of numbers show that the product of their H.C.F. and L.C.M. equals their product:
   (i) 14, 21 (ii) 117, 221
   (iii) 25, 65 (iv) 27, 90

3. Determine the lowest common denominator for fractions in each of the following:

   (i) $\frac{3}{65}, \frac{17}{91}, \frac{5}{117}$ (ii) $2\frac{4}{15}, \frac{7}{135}, \frac{8}{105}$

4. Given that the H.C.F. of two numbers is 16 and their product is 6400, determine their L.C.M.
5. Is the product of three numbers always equal to the product of their H.C.F. and L.C.M.?
6. The H.C.F. and the L.C.M. of two numbers are 13 and 1989, respectively. If one of the numbers is 117, determine the other.
7. Can two numbers have 14 as their H.C.F. and 204 as their L.C.M. Give reasons in support of your answer.
8. In a school there are two sections – Section A and Section B – of Class VI. There are 32 students in Section A and 36 in Section B. Determine the minimum number of books required for their class library so that they can be distributed equally among students of Section A or of Section B.
9. Telegraph poles occur at equal distances of 220 m along a road and heaps of stones are put at equal distances of 300 m along the same road. The first heap is at the foot of the first pole. How far from it along the road is the next heap which lies at the foot of a pole?
10. Find the smallest number which when divided by 25, 40 and 60 leaves remainder 7 in each case.
11. A boy saves Rs 4.65 daily. Find the least number of days in which he will be able to save an exact number of rupees.
12. In a morning walk three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm, respectively. At what distance from the starting point will they again step off together?
13. Determine the two numbers nearest to 10000 which are exactly divisible by each of 2, 3, 4, 5, 6 and 7.
14. Determine the number nearest to 100000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21.

Things to Remember

1. A factor of a number divides the number exactly.
2. A multiple of a number is exactly divisible by the number.
3. Every number is a factor as well as a multiple of itself.
4. 1 is a factor of every number and is the only number, which is neither prime nor composite.
5. 2 is the only even prime number.
6. Prime factorization of a number is unique irrespective of the order of its factors.
7. Primes occurring in pairs with a difference of two are called twin primes.
8. The product of H.C.F. and L.C.M. of two numbers equals their product.
9. Coprime numbers have 1 as their only common factor. Thus the H.C.F. of any two prime or coprime numbers equals 1.
10. The L.C.M. of any two prime or coprime numbers equals their product.
11. The H.C.F. of two or more numbers is never greater than any of the numbers.
12. The L.C.M. of two or more numbers is never less than any of the numbers.
13. The H.C.F. of two or more numbers is a factor of their L.C.M.
14. A number is divisible by
    - (i) 2, if the unit's digit of the number is 0, 2, 4, 6 or 8.
    - (ii) 3, if the sum of the digits is divisible by 3.
    - (iii) 4, if the number formed by its digits in ten's and unit's places is divisible by 4.

(iv) 5, if unit's digit is 0 or 5.
(v) 6, if it is divisible by both 2 and 3.
(vi) 8, if the number formed by its digits in hundred's, ten's and unit's places is divisible by 8.
(vii) 9, if the sum of the digits is divisible by 9.
(viii) 10, if the unit's digit is 0.
(ix) 11, if the difference of the sum of its digits in odd places and the sum of its digits in even places (starting from unit's place) is either 0 or divisible by 11.

CHAPTER FIVE

Integers

IN THIS CHAPTER, we shall extend our number system from whole numbers to integers, represent integers on the number line, discuss the idea of absolute value of an integer, operations on integers and their properties and study what are called powers of integers.

5.1 The Need for Integers and Their Introduction

In the earlier chapters, we have studied whole numbers and operations on them. We saw that it is not always possible to subtract a number from another in whole numbers. For example, we have no whole number to represent $6-10$, $10-17$, etc. Therefore, there is a need to extend our number system so that we have numbers to represent such differences.

Let us consider natural numbers, i.e. 1, 2, 3, 4, 5, . . . and for each natural number (whole number other than zero) we create a new number as follows:

> For 1 we create -1 (called *negative* one or *minus* one), such that $1+(-1)=0$. -1 and 1 are called *opposites* of each other.
> For 2 we create -2 (called negative two or minus two), such that $2+(-2)=0$. -2 and 2 are called opposites of each other.
> For 3 we create -3 (called negative three or minus three), such that $3+(-3)=0$. 3 and -3 are called opposites of each other, and so on.

Thus, our new collection together with whole numbers becomes 0, 1, -1, 2, -2, 3, -3 . . . . We rewrite this as

$$\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$$

These numbers are called *integers.* The numbers 1, 2, 3 . . . , i.e. natural numbers are called *positive integers* and −1, −2, −3 . . . are called *negative integers.*

The number 0 is simply an integer — it is neither positive nor negative. Positive integers are also written as +1, +2, +3 . . ., however, the plus (+) sign is usually omitted and understood.

A point to note here is that whenever opposites are involved simultaneously in a real situation, it is necessary to use integers to express the situation in mathematical terms. For example, *profits* and *losses* of a shopkeeper during certain period, heights of places expressed in terms of distances *above* and *below* sea level, temperatures of objects or places in terms of degrees *above* and below 0°C, etc., all need positive and negative integers for their representation. In such situations, one can represent profits, heights above sea level, temperatures above 0°C, etc. by positive integers and their opposites, namely losses, heights below sea level, temperatures below 0°C, etc. by negative integers.

Remark: We use the symbol '−' to denote negative integers. We also use this symbol to indicate subtraction. The context will make it clear whether we mean negative integer or subtraction. For instance, when we say that the temperature at Srinagar was −8°C, it is immediately clear that no subtraction is involved and that the negative integer '−8' is indicated. On the other hand, when we say, 'determine 46 − 17' it is immediately clear that subtraction of 17 from 46 is required.

5.2 Representation of Integers on Number Line

Since negative integers are opposites of positive integers, viz. natural numbers, we represent them in the opposite direction, i.e. on the left side of zero on the number line. The opposite integers (e.g. 3 and −3) are kept at the same distance from zero which is regarded as neither positive nor negative. Thus, we have the integers on the number line as shown in Fig. 5.1.

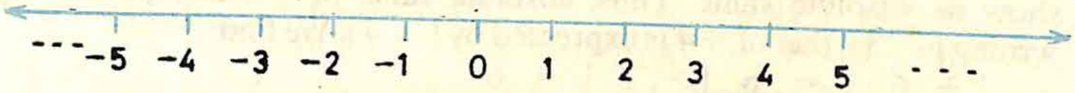


Fig. 5.1

5.3 Ordering of Integers

Let us carefully observe the number line given in Fig. 5.1 above. We recall that every whole number on the right of the number line was regarded as greater than every *whole number* to its left. Here also we follow the same *convention*. We find

$2 > 1$ since 2 is to the right of 1.
$1 > 0$ since 1 is to the right of 0.
$0 > -1$ since 0 is to the right of -1.
$-1 > -2$ since -1 is to the right of -2.

We conclude:

(i) *Every positive integer is greater than every negative integer.*
(ii) *Zero is less than every positive integer.*
(iii) *Zero is greater than every negative integer.*
(iv) *The greater the number, the lesser is its opposite.* For example, $5 > 2$, $-5 < -2$, $-3 > -5$, $3 < 5$. In other words, *If a and b are two integers such that* $a > b$, *then* $-a < -b$.

5.4 Absolute Value of an Integer

The absolute value of an integer is the numerical value of the integer regardless of its sign. Thus, the absolute value of an integer is always positive. On the number line it is regarded as the distance of the integer from 0 irrespective of its direction. Thus, the absolute value of

$+5$ is 5,
-5 is 5,
$+7$ is 7,
-7 is 7,
0 is 0.

The absolute values of integers are essential in the further study of integers. We use two vertical lines, one on either side of the integer to show its absolute value. Thus, absolute value of -3 is expressed by writing $|-3|$, that of $+4$ is expressed by $|+4|$. We find

$$|+6| = 6 \quad |-6| = 6$$
$$|-11| = 11 \quad |+11| = 11$$

We conclude:

If a represents an integer,
$|a| = a$ *if a is positive or zero*
$|a| = -a$ *if a is negative.*

EXERCISE 5.1

1. Give the opposites of
   - (i) Increase in population
   - (ii) Depositing money in a bank
   - (iii) Earning money
   - (iv) Going east
   - (v) 12
   - (vi) -2
2. Indicate the following by using integers:
   - (i) 3° above zero
   - (ii) 5° below zero
   - (iii) A withdrawal of Rs 25 from an account
   - (iv) A deposit of Rs 100 in an account
3. Which number in each of the following pairs is to the right of the other on the number line?
   - (i) 1, 7
   - (ii) $-2, -5$
   - (iii) $0, -3$
   - (iv) $-5, 8$
4. Using the number line, write the integer which is
   - (i) 4 more than 3
   - (ii) 5 less than 2
   - (iii) 8 more than -9
   - (iv) 7 less than -3
5. Which number in each of the following pairs is smaller?
   - (i) $8, -8$
   - (ii) $0, -12$
   - (iii) $-15, -5$
   - (iv) $318, -356$
6. Write all integers between
   - (i) -5 and 2
   - (ii) 0 and 4
   - (iii) -4 and 4
   - (iv) -7 and 0

7. Replace $*$ in each of the following by $<$ or $>$ so that the statement is true:
   - (i) $0 * 5$
   - (ii) $-7 * -17$
   - (iii) $-3 * 0$
   - (iv) $-81 * 18$
   - (v) $-13 * 13$
   - (vi) $-253 * -523$
8. Write the absolute value of each of the following:
   - (i) 17
   - (ii) -23
   - (iii) 0
   - (iv) -107
   - (v) -245
   - (vi) 1024
   - (vii) $(x-2)$ if x is greater than 2
   - (viii) $(x-2)$ if x is less than 2
9. Which of the following statements are true?
   - (i) The smallest integer is zero.
   - (ii) The opposite of zero is zero.
   - (iii) -18 is greater than -5.
   - (iv) A positive integer is greater than its opposite.
   - (v) Zero is not an integer since it is neither positive nor negative.
   - (vi) The absolute value of an integer is always greater than the integer.

5.5 Addition of Integers

We know how to add two whole numbers. Since each whole number is either a positive integer or zero, we can add the integers if they are positive or zero. But what happens if both of them or one of them is negative? We should like to have a rule or rules for addition of integers which would give us answers consistent with our everyday experiences. For example, we know that a profit of Rs 5 and a loss of Rs 2 result in a net profit of Rs 3. Therefore, we must have $5 + (-2) = 3$. A profit of Rs 3 and a loss of Rs 5 result in a net loss of Rs 2. Therefore, we must have $3 + (-5) = -2$. Similarly, a loss of Rs 3 and again a loss of Rs 2 result in a net loss of Rs 5. We, therefore, would like to have $(-3) + (-2) = -5$.

Let us go back to the number line and recall that $+3$ (or 3) represents a movement of 3 units to the right of zero and -3 represents a movement of 3 units to the left. With this background, let us represent some addition facts for integers on the number line.

Let us represent '$2 + 4$' on the number line. We begin at zero and move 2 units to the right. We reach at 2. We further move 4 units to the right and reach at 6 (see Fig. 5.2).

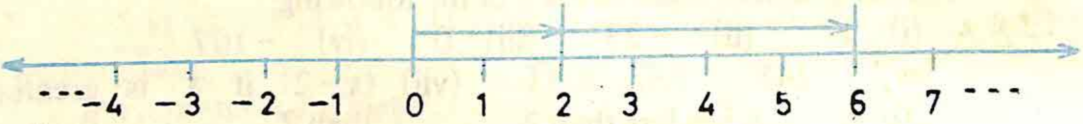


Fig. 5.2

Thus, $2 + 4 = 6$. We note that $|2| + |4| = 6$

Let us now represent '$2 + 4$' on the number line. We first move 2 units to the left of zero and then from there move 4 units to the right to account for $+ 4$ (see Fig. 5.3). We reach at 2.

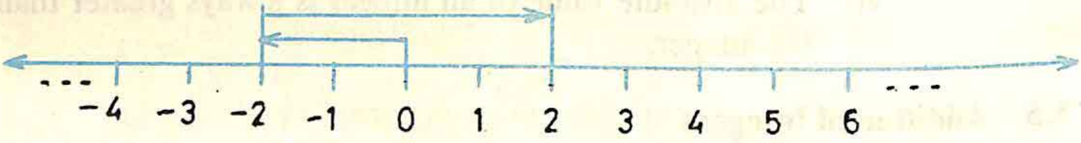


Fig. 5.3

Thus, $-2 + 4 = 2$. We note that $|4| - |-2| = 2$.

Finally, we represent $-2 + (-4)$ on the number line. We first move 2 units to the left of zero to account for -2 and further move 4 units to the left to account for -4. Thus, we reach at -6 (see Fig. 5.4). Therefore $(-2) + (-4) = -6$. We note that $|-2| + |-4| = 6$ and $-(|-2| + 1 -4|) = -6$.

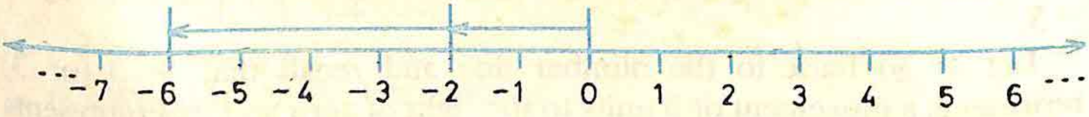


Fig. 5.4

In view of the above, we conclude:

(i) *To add two positive integers or two negative integers, we add their absolute values and assign the sign of the addends to the sum.*

Example 1: Add -2578 and -636.

Solution: We observe that both the integers are negative. Therefore, we use rule (i) mentioned above.

$|-2578| = 2578 \qquad |-636| = 636$

$-2578 + (-636) = -(2578 + 636) = -3214$

(ii) *To add a positive and a negative integer, we determine the difference of their absolute values and assign the sign of the addend having greater absolute value.*

Example 2: Add 384 and -905.

Solution: The integers are of unlike signs. Therefore, to find their sum we use rule (ii), mentioned above.

$|384| = 384 \quad |-905| = 905$

$$\begin{aligned} 384 + (-905) &= -(905 - 384) \\ &= -521 \end{aligned}$$

5.6 Properties of Addition of Integers

When we extend our number system from whole numbers to integers, all properties of operations on whole numbers remain intact. There may be one or two additional properties. It is so because all whole numbers are also integers. We briefly state the properties of addition as follows where a, b, c represent integers.

Property I: *$a + b$ is an integer.*

Examples: (i) $2 + (-7) = -5$, -5 is an integer

(ii) $-3 + (-8) = -11$, -11 is an integer.

(iii) $17 + (-6) = 11$, 11 is an integer.

Property II: $a + b = b + a$

Examples: (i) $3 + (-4) = (-4) + 3$ since each equals (-1).

(ii) $(-3) + (-9) = (-9) + (-3)$ since each equals (-12).

Property III: $a + (b + c) = (a + b) + c$

Examples: (i) $[(-3) + (-2)] + 5 = (-3) + [(-2) + 5]$

Since $[(-3) + (-2)] + 5 = (-5) + 5 = 0$

and $(-3) + [(-2) + 5] = (-3) + 3 = 0$

(ii) $[(-1) + 4] + (-6) = (-1) + [4 + (-6)]$

Since $[(-1) + 4] + (-6) = 3 + (-6) = -3$

and $(-1) + [4 + (-6)] = (-1) + (-2) = -3$

As before, we usually write $a + b + c$ for the equal sums mentioned above.

In view of Properties II and III *even if we rearrange a given collection of three or more integers their sum is not changed.*

Property IV: $a + 0 = 0 + a = a$

Property V: We recall that if we add 1 to any whole number, we get the successor of that whole number. In case of integers also the same property holds. However, in case of whole numbers, 0 was not the successor of any whole number. In the case of integers, 0 is the successor of -1, -1 is the successor of -2, and so on.

Example 3: Find the sum of -21, -9, 63, -22 and -28.

Solution: $(-21) + (-9) + 63 + (-22) + (-28).$

$= [(-21) + (-9)] + 63 + [(-22) + (-28)]$

$= -30 + 63 + (-50)$

$= [-30 + (-50)] + 63$

$= -80 + 63$

$= -17$

EXERCISE 5.2

1. Draw a number line and represent each of the following on it:

(i) $-6 + 8$ (ii) $5 + (-9)$

(iii) $-3 + (-8)$ (iv) $-1 + (-2) + 2$

(v) $-2 + 7 + (-8)$ (vi) $-2 + (-3) + (-5)$

2. Add the integers in each of the following:

(i)	$-245, 111$	(ii)	$2567, -3$
(iii)	$10001, -2$	(iv)	$-99005, 360$
(v)	$-498, -320$	(vi)	$-5894, 0$
(vii)	$3003, -999$	(viii)	$2884, -2884$
(ix)	$2547, -2548$	(x)	$-623, -5832, 623$

(xi) $-982, 1934, -18, -2034$

(xii) $-4329, 4648, 4371$

3. Find the sum:

(i) $100 + (-66) + (-34)$

(ii) $1262 + (-366) + (-962) + 566$

(iii) $908 + (-8) + (-1) + 1 - 300$

(iv) $-391 + (-81) + 9 + (-18)$

(v) $373 + (-245) + (-373) + 145 + 3000$

(vi) $1 + (-475) + (-475) + (-475) + (-475) + 1900$

4. Let us invent an operation $*$ for integers such that for two integers a and b

$$a * b = a + b + (-1)$$

For example, $2 * 3 = 2 + 3 + (-1) = 4$. Determine:

(i) $3 * (-4)$ (ii) $15 * (-1)$

5. Which of the following statements are true?

(i) The sum of a number and its opposite is zero.

(ii) The sum of two negative numbers is a positive number.

(iii) The sum of a negative number and a positive number is always a negative number.

(iv) The successor of -297 is -298.

(v) The sum of three different integers can never be zero.

5.7 Subtraction of Integers

The positive and negative integers suggest pairs of opposite numbers, for instance, 1 and -1, 2 and -2, 3 and -3, etc. We observe that the sum in each pair is zero, namely, $1 + (-1) = 0$, $2 + (-2) = 0$, etc. In any such pair each integer is called the *negative* (or *additive inverse*) of the other.

Thus, *for each non-zero integer, a, there is an integer '$-a$' such that $a + (-a) = 0$. '$-a$' is called the negative (or additive inverse) of a. Negative of zero is zero itself.*

We recall that subtraction is a process inverse to that of addition. For instance, to subtract 3 from 7 is the same as to find a number which added to 3 gives 7.

Suppose we want to subtract '-3' from 4. It means we want to find a number which added to '-3' gives 4. Obviously, such a number is 7. Let us see how we do it with the help of number line. To subtract '-3' from 4 suggests that if we are 3 steps to the left of starting point, i.e. 0, we need to determine the number of steps required to reach 4. From Fig. 5.5, we see that the number of steps required is 7. Therefore,

$$4 - (-3) = 7$$

$$\text{Also, } 4 + 3 = 7$$

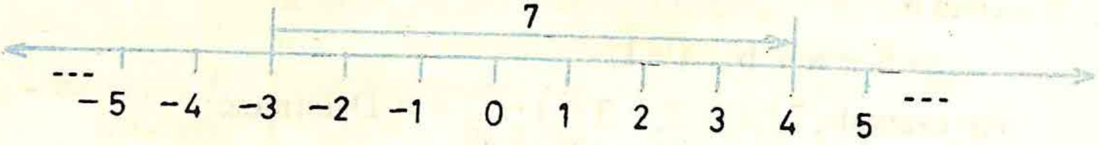


Fig. 5.5

Thus, we see that to subtract '-3' from 4, we add the negative (or additive inverse) of '-3' to 4. In fact this is the rule for subtraction of integers. The rule states:

If a and b are two integers, to subtract b from a, we change the sign of b and add it to a, i.e.

$$a - b = a + (-b)$$

Thus, we use $+(-b)$ and $-b$ interchangeably.

Example 4: Subtract '-13' from '-5'.

Solution: We change the sign of '-13' and add to '-5'.

Thus, $-5 - (-13) = -5 + 13 = 8$

In view of the above discussions and various properties of addition we are now in a position to find the value of an expression containing various terms with plus and minus signs as follows:

Step 1: *We add all terms with plus signs together.*

Step 2: *We add all terms with minus signs together.*

Step 3: *We find the difference of the absolute values of the two sums obtained in Steps 1 and 2.*

Step 4: *We assign to the result of Step 3 the sign of the sum having larger absolute value.*

Example 5: Find the value of

$$-12 + (-98) - (-84) + (-7)$$

Solution: We rewrite the given expression as

$$\begin{aligned} -12 - 98 + 84 - 7 &= (-12 - 98 - 7) + 84 \\ &= -117 + 84 \\ &= -33 \end{aligned}$$

5.8 Properties of Subtraction

Property I: We have seen that the difference of any two integers is also an integer. In other words, *if a and b are two integers and $a - b = c$, then c must be an integer.*

Note that this property does not hold for whole numbers.

Property II: In whole numbers 0 had no predecessor. In integers, -1 is the predecessor of 0. Thus, if a is any integer, '$a-1$' is its predecessor.

Property III: As in the case of whole numbers, if a is an integer, $a-0 = a$.

EXERCISE 5.3

1. Subtract the first integer from the second in each of the following:
   (i) 3, 8 (ii) 10, -4 (iii) -15, 10 (iv) -200, -100

(v) $1001, 101$ (vi) $2, -7$ (vii) $-812, 3126$
(viii) $8650, \quad -6$ (ix) $-3987, -4109$ (x) $-155, \quad 0$
(xi) $0, -1005$ (xii) $83241, 40321$

2. Subtract -5 from 7. Subtract 7 from -5. Are the two results the same?
3. Replace $*$ by '$<$' or '$>$' in each of the following to make the statement true:
   (i) $(-6) + (-9) * (-6) - (-9)$
   (ii) $(-12) - (-12) * (-12) + (-12)$
   (iii) $(-20) - (+20) * 20 - (+65)$
4. Fill in the blanks:
   (i) $-6 + \_\_\_ = 0$
   (ii) $19 + \_\_\_ = 0$
   (iii) $12 + (-12) = \_\_\_$
   (iv) $-4 + \_\_\_ = 12$
   (v) $-256 + \_\_\_ = -396$
   (vi) $\_\_\_ - 215 = -64$
5. The sum of two integers is 48. If one of the integers is -24, determine the other.
6. The sum of two integers is -396. If one of them is 64, determine the other.
7. Find the value of
   (i) $-17 - (-13)$
   (ii) $-7 - 8 - (-25)$
   (iii) $(2 - 3) + (2 - 3)$
   (iv) $-13 + 32 - 18 - 1$
   (v) $50 - (-48) - (-2)$
   (vi) $-7 + (-8) + (-90)$
   (vii) $18 - [(-3) + 15]$
   (viii) $-12 - [(-15) + (-2) - 3]$
8. p and q are two integers such that p is the predecessor of q. Find the value of $P - q$.
9. If Δ is an operation such that for integers a and b, $a \Delta b = -a + b - (-2)$, find the value of

(i) $4 \Delta 3$ (ii) $(-2) \Delta (-3)$ (iii) $6 \Delta (-5)$ (iv) $(-5) \Delta 6$

10. Which of the following statements are true?
    (i) $-13 > -8 - (-2)$
    (ii) $-4 + (-2) < 2$
    (iii) The negative of a negative integer is a positive integer.
    (iv) If a and b are two integers such that a > b then a − b is always a positive integer.

11. Calculate
    $1 - 2 + 3 - 4 + 5 - 6 + \ldots + 19 - 20$

12. Calculate the sum
    $2 + (-2) + 2 + (-2) + 2 + (-2) + \ldots$
    (i) if the number of twos (terms) is 319.
    (ii) if the number of twos is 230.

5.9 Multiplication of Integers

We recall that multiplication is repeated addition. For instance,
$3 \times 5 = 5 + 5 + 5$ or 15.

In the same way we can find the product of any two integers. Thus,

$$\begin{aligned} 4 \times (-5) &= (-5) + (-5) + (-5) + (-5) \\ &= -20 \\ &= -(4 \times 5) \end{aligned}$$

Let us now consider the product $(-4) \times 3$. We have

$$\begin{aligned} (-4) \times 3 &= 3 \times (-4) \qquad \text{(Why?)} \\ &= -4 + (-4) + (-4) \\ &= -12 \\ &= -(3 \times 4) \end{aligned}$$

In view of the above, we conclude:

To find the product of a positive and a negative integer, we find the product of their absolute values and assign minus sign to the product.

Let us now find the product of two integers with like signs, i.e. either both the integers are positive or both of them are negative.

If both the integers are positive, they are whole numbers and the product is obtained as in the case of whole numbers.

Let us consider the case when both the integers are negative. Let the integers be '-3' and '-2'. To do this, let us observe the following table:

$$\begin{aligned}(-3)\times 4 &= -12\\ (-3)\times 3 &= -9\\ (-3)\times 2 &= -6\\ (-3)\times 1 &= -3\\ (-3)\times 0 &= 0\\ (-3)\times(-1) &= ?\end{aligned}$$

We observe that as the second integer decreases by 1, the product increases by 3. Therefore, the product $(-3)\times(-1)$ should be 3 more than 0, i.e. 3. Similarly, $(-3)\times(-2)$ should be 3 more than 3, i.e. 6, and so on. We have, thus, found

$$\begin{aligned}(-3)\times(-2) &= 6\\ &= +(3\times 2)\end{aligned}$$

The distributive property of multiplication over addition (which also holds good in integers) also helps us in getting the same result. We have

$$(-3)\times[2(-2)] = (-3)\times(0)$$

$$\text{or, } (-3)\times 2 + (-3)\times(-2) = 0$$

$$\text{or, } -6 + (-3)\times(-2) = 0$$

This shows that $(-3)\times(-2)$ is the integer which when added to '-6' gives the sum zero. We know that 6 added to '-6' gives zero as the sum. Therefore,

$$(-3)\times(-2) = 6 = +(3\times 2)$$

In view of the above, we conclude:

The product of two integers, both positive or both negative, is a positive integer equal to the product of the absolute values of the integers.

Example 6: Determine each of the following products:

(i) $(-15)\times(-18)$ (ii) $(-23)\times 7$

Solution: (i) $(-15)\times(-18) = +(15\times 18)$

$= 270$

(ii) $(-23)\times 7 = -(23\times 7)$

$= -161$

5.10 Properties of Multiplication

All properties of multiplication of whole numbers also hold *for* integers. We restate the properties in brief as follows wherein a, b, c are integers.

Property I: $a \times b$ *is an integer*

Examples: (i) $2 \times (-3) = -6$, -6 is an integer.

(ii) $(-3) \times (-7) = 21$, 21 is an integer.

Property II: $a \times b = b \times a$

Examples: (i) $3 \times (-4) = (-4) \times 3$ as each equals (-12).

(ii) $(-6) \times (-7) = (-7) \times (-6)$ as each equals 42.

Property III: $a \times (b \times c) = (a \times b) \times c$

Examples: (i) $(-2) \times [3 \times (-5)] = [(-2) \times 3] \times (-5)$

Since $(-2) \times [3 \times (-5)] = (-2) \times (-15) = 30$

and $[(-2) \times 3] \times (-5) = (-6) \times (-5) = 30$

(ii) $(-4) \times [(-5) \times (-6)] = [(-4) \times (-5)] \times (-6)$

Since $(-4) \times [(-5) \times (-6)] = (-4) \times (30) = -120$

and $[(-4) \times (-5)] \times (-6) = (20) \times (-6) = -120$

As before, we write $a \times b \times c$ for the equal products mentioned above.

In view of Properties II and III *even if we rearrange the integers in a product of 3 or more integers, it does not change the product.*

We also observe that

(i) When the number of negative integers in a product is odd, the product is negative.

(ii) When the number of negative integers in a product is even, the product is positive.

For example, each of the products $(-3) \times (-2) \times (-3) \times 4$, $(-1) \times 7 \times 8$ and $(-4) \times (-5) \times (-6) \times (-7) \times (-3)$ must be negative and each of the products $(-2) \times (-5)$, $(-2) \times (-6) \times 3$ and $(-7) \times (-8) \times (-5) \times (-9) \times 7 \times 4$ must be positive.

Property IV: $a \times 0 = 0 \times a = 0$

Property V: $a \times 1 = 1 \times a = a.$

We have the following additional property.

Property VI: $a \times (-1) = (-1) \times a = -a$

We know that a and $-a$ are opposties of each other. Thus, to find the negative of an integer, we need to multiply the integers by (-1).

Property VII: (i) $a \times (b + c) = a \times b + a \times c$

(ii) $a \times (b - c) = a \times b - a \times c$

Examples: (i) $(-3) \times [(-5) + 4] = (-3) \times (-5) + (-3) \times 4$

Since $(-3) \times [(-5) + 4] = (-3) \times (-1) = 3$

and $(-3) \times (-5) + (-3) \times 4 = 15 - 12 = 3$

(ii) $(-6) \times [(-2) - 5] = (-6) \times (-2) - (-6) \times 5$

Since $(-6) \times [(-2) - 5] = (-6) \times (-7) = 42$

and $(-6) \times (-2) - (-6) \times 5 = 12 - (-30) = 42$

EXERCISE 5.4

1. Find each of the following products:

(i) $2 \times (-15)$
(ii) $(-225) \times 8$
(iii) $(-17) \times (-20)$
(iv) $3 \times (-8) \times 5$
(v) $9 \times (-3) \times (-6)$
(vi) $(-12) \times (-12) \times (-12)$
(vii) $(-2) \times 36 \times (-5)$
(viii) $(-8) \times (-43) \times 0$
(ix) $18 \times (-185) \times (-4)$
(x) $(-45) \times 55 \times (-10)$
(xi) $(-1) \times (-2) \times (-3) \times (-4) \times (-5)$
(xii) $(-3) \times (-6) \times (-9) \times (-12)$

2. Complete the following Multiplication Table:

Second Number

First Number

X	− 4	− 3	− 2	− 1	0	1	2	3	4
− 4									
− 3									
− 2									
− 1									
0									
1									
2									
3									
4									

Is the Multiplication Table symmetrical about the diagonal joining the upper left corner to the lower right corner?

3. Find the value of
   - (i) $(-8) \times 0 \times 37 \times (-37)$
   - (ii) $1569 \times 887 - 569 \times 887$
   - (iii) $(-183) \times (-44) + (-183) \times (-56)$
   - (iv) $18946 \times 99 - (-18946)$
   - (v) $15625 \times (-2) + (-15625) \times 98$
   - (vi) $(-8) \times (10 - 5 - 43 + 98)$
4. What will be the sign of the product if we multiply together
   - (i) 8 negative integers and 1 positive integer?
   - (ii) 21 negative integers and 3 positive integers?
5. Determine the integer whose product with '− 1' is
   (i) − 40 (ii) 46 (iii) 0

6. Compare, i.e. state which is greater:
   - (i) $(8 + 9) \times 10$ and $8 + 9 \times 10$
   - (ii) $(8 - 9) \times 10$ and $8 - 9 \times 10$
   - (iii) $[(-2) - (5)] \times (-6)$ and $(-2) - 5 \times (-6)$
7. Which of the following statements are true?
   - (i) The product of three negative integers is a negative integer.
   - (ii) Of the two integers, if one is negative, then their product must be negative.
   - (iii) The product of a negative and a positive integer may be zero.
   - (iv) There does not exist an integer b such that for $a > 1$
     $$a \times b = b \times a = b$$
   - (v) For all non-zero integers a and b, $a \times b$ is always greater than either a or b.

5.11 Division of Integers

We know that division is inverse of multiplication. Each multiplication fact gives rise to corresponding division fact(s). For example, the multiplication fact '$2 \times 3 = 6$' gives the following division facts:

$$6 \div 2 = 3 \text{ and } 6 \div 3 = 2$$

In the case of integers we have $(-4) \times 5 = -20$ and the corresponding division facts are:

$$-20 \div (-4) = 5 \quad \text{and} \quad -20 \div 5 = -4$$

As usual, the number to be divided is called *dividend* and the number which divides is called the *divisor.* The result of division is called *quotient.* In the above example, we note that *when dividend is negative and divisor is negative, the quotient is positive. When the dividend is negative and divisor is positive, the quotient is negative.* Let us consider one more example and see whether or not the same thing happens. We take the multiplication fact $(-5) \times (-8) = 40$. We get

Corresponding Division Fact	*Dividend*	*Divisor*	*Quotient*
$40 \div (-5) = -8$	Positive	Negative	Negative
$40 \div (-8) = -5$	Positive	Negative	Negative

In view of the above we conclude:

(i) *The quotient of two integers both positive or both negative is a positive integer equal to the quotient of the corresponding absolute values of the integers.*

(ii) *The quotient of a positive and a negative integer is a negative integer and its absolute value is equal to the quotient of the corresponding absolute values of the integers.*

As usual, the divisor is a non-zero integer. In other words, *division by zero is not defined.*

Example 7: Divide

(i) 68 by -17 (ii) -78 by 13 (iii) (-60) by (-12)

Solution: (i) $|68| = 68, |-17| = 17$

We have $68 \div 17 = 4$

Therefore, $68 \div (-17) = -4$

(ii) $(-78) \div 13 = -(78 \div 13) = -6$

(iii) $(-60) \div (-12) = +(60 \div 12) = 5$

5.12 Properties of Division

Property I: *If a and b are two integers, a* $\div$ b is not always an integer. For example, none of $(-4) \div 5$, $5 \div (-3)$, $1 \div 2$ is an integer.

Property II: For any integer a,

$a \div a = 1 \ (a \neq 0) \quad a \div 1 = a$

Property III: For any integer a,
$a \div (-1) = -a, a \div (-a) = -1, (a \neq 0)$
$(-a) \div a = -1 \ (a \neq 0)$

Property IV: For any non-zero integer a, $0 \div a = 0$.

EXERCISE 5.5

1. Find the quotient in each of the following:

(i) $18 \div (-3)$
(ii) $(-18) \div 3$
(iii) $(-18) \div (-3)$
(iv) $36 \div (-9)$
(v) $(-48) \div (-16)$
(vi) $0 \div (-12)$
(vii) $(-1728) \div 12$
(viii) $-15625 \div (-125)$
(ix) $(-729) \div (-81)$
(x) $10569 \div (-1)$
(xi) $200000 \div (-100)$.
(xii) $17699 \div (-17699)$

2. Fill in the blanks:

(i) $296 \div$ —— $= 296$
(ii) $-3785 \div$ —— $= 1$
(iii) —— $\div 578 = 0$
(iv) —— $\div 1 = -3065$
(v) —— $\div 156 = -2$
(vi) —— $\div 567 = -1$

5.13 Use of Brackets

In order to simplify a numerical expression with two or more of the fundamental operations, we follow the conventions given below:

(i) *Generally we perform operations sequentially from left to right in the order division, multiplication, addition, subtraction.*

For example,

(a) $36 \div 6 + 3 = 6 + 3 = 9$

(b) $12 - 4 \div 2 = 12 - 2 = 10$

(c) $55 - 7 \times 3 + 35 - 4$

$= 55 - 21 + 35 - 4$

$= 55 + 35 - 21 - 4$

$= 90 - 21 - 4$

$= 69 - 4$

$= 65$

(ii) When the operations of division and multiplication occur simultaneously (as in $36 \div 6 \times 3$) without an operation of addition or subtraction between them, which of the two operations, multiplication or division, is to be performed first, will be indicated by ( ). Thus, $36 \div (6 \times 3)$ shows that first the operation of multiplication is to be performed.

(iii) Many a time we come across situations where we need to be certain to indicate definitely which operations are to be performed first. To do this we use brackets (grouping symbols). For example, if we want to divide 36 by the sum of 2 and 4, we indicate the sum of 2 and 4 by $(2 + 4)$ and write it as

$$36 \div (2 + 4) = 36 \div 6 = 6$$

If we do not use the symbol ( ), the expression becomes $36 \div 2 + 4$ which means $18 + 4$, i.e. 22. In many situations arrangement of terms within a second group has to be indicated. In such a case, we use different symbols so that we can easily know which terms are combined together. For example, when we say, '114 divided by the number 1 less than the product of 4 and 5', we treat '4×5' as one entity. We write (4×5) for this entity. We treat '$(4 \times 5) - 1$' as the second entity and write $\{(4 \times 5) - 1\}$ for this using another symbol '{ }' to indicate the new arrangement. Thus the expression becomes

$$114 \div \{(4 \times 5) - 1\}$$

Most commonly used grouping symbols are:

Symbols	*Names*
( )	Parentheses or common brackets
{ }	Braces or curly brackets
[ ]	Brackets or square brackets

The left part of each symbol indicates its start and the right part indicates its end.

(iv) In some cases we may use another symbol '—' called *bar/ vinculum* which in order comes first. It occurs on the innermost terms and is placed like

$$36 \div (8 - \overline{4+2}).$$

In an expression, if more than one symbols are used, we first remove the innermost symbol by doing the operations involved and then proceed with the next innermost, and so on. While removing such symbols we keep in mind that we mean 'multiplication' when there is no sign of operation between a number and a grouping symbol. Thus,

$$4(6-2) = 4 \times (6-2)$$

If an arrangement within brackets is preceded by a '+' sign, to remove brackets, we do the operations as follows:

$$\begin{aligned} 9 + (7-3+2) &= 9 + 7 - 3 + 2 \\ &= 18 - 3 \\ &= 15 \end{aligned}$$

If an arrangement within brackets is preceded by a '−' sign and we want to remove brackets, then we change all the positive signs within brackets to negatives and vice versa. For example,

$$\begin{aligned} 9 - (7 - 3 + 2) &= 9 - 7 + 3 - 2 \\ &= 12 - 7 - 2 \\ &= 3 \end{aligned}$$

Let us now consider some examples that will make these ideas more clear.

Example 8 : Find the value of

$$17 - [3 + \{18 - (19 - 2)\}]$$

Solution : $17 - [3 + \{18 - (19 - 2)\}]$

$= 17 - [3 + \{18 - 17\}]$

$= 17 - [3 + 1]$

$= 17 - 4$

$= 13$

Example 9: Simplify:

$(-13) + (-6) \div 2 - [(-5) \times (-4) - \{2 - (3 - 5)\}]$

Solution: The given expression is equal to

$(-13) + (-3) - [20 - \{2 - (-2)\}]$

$= (-13) + (-3) - [20 - 4]$

$= (-16) - (16)$

$= -32$

EXERCISE 5.6

1. Find the value of
   - (i) $24 + 15 \div 3$
   - (ii) $120 - 20 \div 2$
   - (iii) $12 - (3 \times 5) + 3$
   - (iv) $3 - (5 - 6 \div 3)$
   - (v) $28 - 5 \times 6 + 2$
   - (vi) $36 \div (5 + 7)$
   - (vii) $(-15) + 4 \div (5 - 3)$
   - (viii) $(-2) + (-8) \div (-4)$
   - (ix) $17 + (-3) \times (-5) - 6$
   - (x) $2 \div \overline{3 - 2}$
   - (xi) $(-40) \times (-1) + (-28) \div 7$
   - (xii) $(-5) - (-48) \div (-16) + (-2) \times 6$
2. Using brackets, write a mathematical expression for each of the following:
   - (i) Five multiplied by the sum of two and three
   - (ii) Twelve divided by the sum of one and three
   - (iii) Twenty divided by the difference of seven and two
   - (iv) Eight subtracted from the product of two and three
   - (v) Forty divided by one more than the sum of nine and ten
   - (vi) Two multiplied by one less than the difference of nineteen and six

3. Simplify:

(i) $20 + \{10 - 5 + (7 - 3)\}$

(ii) $81 \times [59 - \{7 \times 8 + (13 - 2 \times 5)\}]$

(iii) $121 \div [17 - \{15 - 3(7 - 4)\}]$

(iv) $3[18 + \{3 + 4(4 - 2)\}]$

(v) $(14 - 7) \times [8 + \{3 + \overline{7 - 1}\}]$

(vi) $18 + \{1 + (15 - 2) \times 4\}$

(vii) $2 - [2 - \{2 - (2 - \overline{2 - 2})\}]$

(viii) $118 - [121 \div (11 \times 11) - (-4) - \{3 - \overline{9 - 2}\}]$

(ix) $(-1)\{(-5) + (-25)\} \times (-7) - (8 - 10)(-4)$

(x) $15 - (-3)\{4 - \overline{7 - 3}\} \div 3[5 + (-3) \times (-6)]$

5.14 Powers of Integers

In mathematics we have to deal with a number of problems involving multiplication of a number by itself several times. We use a convenient notation to indicate this.

For	*We Write*	*We Read*
2×2	2^2	Two raised to two or two squared or second *power* of two
$2 \times 2 \times 2$	2^3	Two raised to three or two cubed or third power of two
$2 \times 2 \times 2 \times 2$	2^4	Two raised to four or fourth power of two
$2 \times 2 \times 2 \times 2 \times 2$	2^5	Two raised to five or fifth power of two

Thus, $2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32$, and so on. Obviously, $2^1 = 2$. In '2^5' the number '2' is called the *base* and '5' the *exponent*. 2^5, i.e. 32 is the *fifth power* of 2.

Similarly, $(-3)^2 = (-3) \times (-3) = 9$

$(-3)^3 = (-3) \times (-3) \times (-3) = -27$

$(-3)^4 = (-3) \times (-3) \times (-3) \times (-3) = 81$

and so on. Obviously, $(-3)^1 = -3$. It is easy to verify that

$$(-1)^{\text{odd positive integer}} = -1$$
$$(-1)^{\text{even positive integer}} = 1$$

Note: If a is an integer, a^2 is called a *perfect square.* Since $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, etc. the numbers 1, 4, 9, 16, etc. are perfect squares. We note $(-1)^2 = 1$, $(-2)^2 = 4$, $(-3)^2 = 9$, $(-4)^2 = 16$, etc.

If a is an integer, a^3 is called a *perfect cube.* Since $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, etc. the numbers 1, 8, 27, 64, etc. are perfect cubes. We note $(-1)^3 = -1$, $(-2)^3 = -8$, $(-3)^3 = -27$, $(-4)^3 = -64$, etc.

EXERCISE 5.7

1. Give the base and exponent for each of the following:
   (i) 5^7 (ii) $(-2)^3$ (iii) 1^1
   (iv) $(-6)^1$ (v) $(-27)^2$ (vi) 10^5
2. Write, using power notation:
   (i) $10 \times 10 \times 10 \times 10$
   (ii) $(-13) \times (-13) \times (-13) \times (-13) \times (-13) \times (-13)$
3. Find the value of
   (i) 50^2 (ii) $(-1)^{51}$ (iii) 1^{100} (iv) $(-1)^{20}$
   (v) $(-2)^8$ (vi) $2^3 \times 3^2$ (vii) $2^3 \times 2^5$
   (viii) $(-2)^6 \div (-2)^2$
   (ix) $(-4)^5 \div (-4)^2$ (x) $(-2)^4 \times (-3)^3 \times (-1)$
   (xi) $(-1)^3 \times (-10)^2$ (xii) $2^3 \times (-3)^2 \times 8$
4. Find the squares of first ten natural numbers. Observe their units' digits. What do you note?
5. Find the cubes of first ten natural numbers.
6. Find:
   (i) 20^2 (ii) 100^2 (iii) 200^2 (iv) 70^2 (v) 150^2 (vi) 1000^2

7. Find the cube of each of the following:

   (i) -12 (ii) -13 (iii) -15 (iv) 11 (v) 100 (vi) 1000

8. Verify each of the following:

   (i) $(-2)^4 \times (-2)^3 = (-2)^7$

   (ii) $10^2 \times 10^3 = 10^5$

   (iii) $(-4)^5 \div (-4)^2 = (-4)^3$

   (iv) $3^7 \div 3^2 = 3^5$

9. Which of the following statements are true?

   (i) The difference between 6^5 and 5^6 is zero.

   (ii) Square of any integer is positive.

   (iii) Cube of a negative integer is negative.

   (iv) $3^6 \div 3^5 = 3^{6-5}$

   (v) $2^3 \times 2^4 = 2^{3+7}$

Things to Remember

1. Every positive integer is greater than every negative integer.
2. Zero is less than every positive integer and greater than every negative integer.
3. The greater the number, the lesser is its opposite (additive inverse).
4. The absolute value of an integer is the numerical value of the integer without regard to its sign. Thus, for integer a,

   $|a| = a$ if a is positive or zero.

   $|a| = -a$ if a is negative.
5. The sum of two negative integers is a negative integer having the absolute value equal to the sum of the absolute values of the given integers.

6. To find the sum of a positive and a negative integer, we determine the difference of their absolute values and assign the sign of the addend having greater absolute value.
7. All properties of operations on whole numbers also hold in case of integers. In addition we have the following properties:
   (i) If a and b are integers, '$a - b$' is always an integer.
   (ii) For any integer a, $a \times (-1) = (-1) \times a = -a$.
   (iii) There does not exist the smallest integer.
8. To subtract an integer b from an integer a, we change the sign of b and add it to a.
9. To find the product (or quotient) of a positive and a negative integer, we find the product (or quotient) of their absolute values and assign minus sign to the product (or quotient).
10. The product (or quotient) of two integers both positive or both negative is a positive integer equal to the product (or quotient) of the absolute values of the integers.
11. To remove grouping symbols from an expression, we first remove the innermost grouping symbol, next the innermost symbol of all that remain, and so on.
12. $(-1)^{\text{odd number}} = -1$, $(-1)^{\text{even number}} = 1$.

UNIT TWO

ALGEBRA

Algebra is often called the shorthand of mathematics because in it statements, instructions and results can be presented in a very concise form. If we look back to the pages of history, the word 'algebra' is derived from the title of the book *Aljebar w' al almugabalah*, written about A.D. 825 by an Arab mathematician, Mohammed ibn Al Khowarizmi of Baghdad. You will read about it later on.

History tells us that the credit of using symbols for numbers goes back to the Ahmes Papyrus, written about 1500 B.C., in which an unknown number is designated by the word 'hau', meaning a heap.

Ancient Indian mathematicians made copious use of symbols to denote unknown quantities. They gave various names such as यावत-तावत (Yavat-tavat) (meaning 'so much as') or वर्ण (Varna), बीज (Bija), etc. to unknowns and also used the first letters का, नी, पी (Ka, Nee, Pee), etc. of the names of colours such as काला (black), नीला (blue), पीला (yellow), to denote them. The use of these letters and the method of raising them to different exponents were quite common in about 300 B.C.

Great Indian mathematicians, Aryabhatta (born in A.D. 476), Brahmagupta (born in A.D. 598), Mahavira (around A.D. 850), Sridhara (around A.D. 1025), and Bhaskara II (born in A.D. 1114) contributed a lot to the study of algebra. The following problem from the famous work of Bhaskara II, *Lilawati*, may be given as an example:

> Out of a swarm of bees, one-fifth part settled on a blossom of Kadamba, and one-third on a flower of Silindhri, three times the difference of those numbers flew to the bloom of a Kutaja. One bee, which remained, hovered and flew about in the air, allured at the same moment by the pleasing fragrance of a Jasmine and Pandanus. Tell me, charming woman, the number of bees.

So far we have dealt with the numbers of arithmetic. We have also seen how these numbers can be put in general forms, with letters like a, b, c, etc. We have read about operations with numbers and also with letters representing them. We would now like to carry on further without referring at each step the numbers of letters they represent. In fact, we shall now work with symbols rather than letters and perform such operations like addition, subtraction, multiplication, etc. This is precisely what we do in algebra.

The operations of addition, subtraction, multiplication and division on these letter symbols and powers on them are taken up in Chapter Six. In the same chapter, algebraic expressions and their addition and subtraction will also be discussed. We take up an important aspect of algebra in Chapter Seven. Simple linear equations in one variable are introduced and some rules have been explained to solve these equations in this chapter.

CHAPTER SIX

Algebraic Expressions

IN THIS CHAPTER, we shall make a beginning of the study of algebra. Here we shall discuss the idea of an algebraic expression, its terms, evaluation, etc. Addition and subtraction of the algebraic expressions will also be discussed.

6.1 Use of Letters to Denote Numbers

So far, we have been mostly using number symbols (or numerals) 0, 1, 2, 3, etc. to represent numbers and the signs of operations of addition ($+$), subtraction ($-$), multiplication ($\times$) and division ($\div$) to do many arithmetical calculations. Consider the following examples:

(i) Let us examine the squares shown in Fig. 6.1.

The perimeter of the square [Fig. 6.1(i)] of side 3 units is $(3 + 3 + 3 + 3)$ units $= 12$ units or 4×3 units. The perimeter of the square [Fig. 6.1(ii)] of side 4 units is $(4 + 4 + 4 + 4)$ units $= 16$ units or 4×4 units. Similarly, $(6 + 6 + 6 + 6)$ units $= 24$ units or 4×6 units is the perimeter of the square given in Fig. 6.1 (iii). We note that in each case the perimeter is four times the length of its sides, i.e.

Perimeter $= 4 \times$ (length of the side)

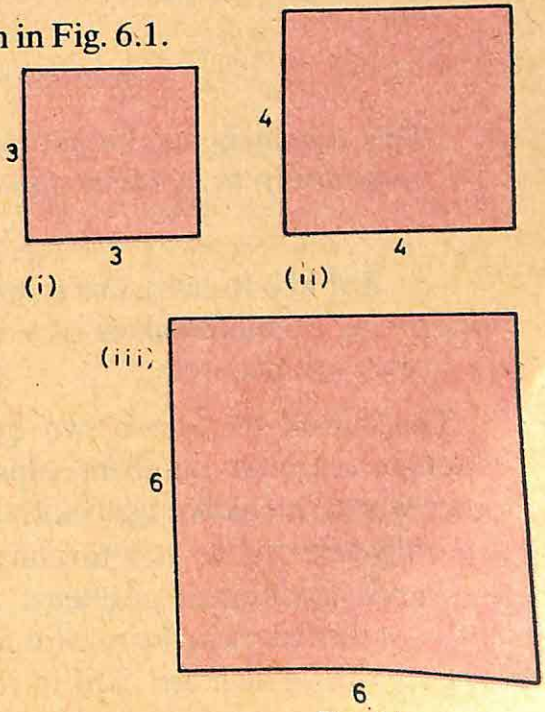


Fig. 6.1

unless otherwise stated, instead of saying 'x represents a number', we shall simply say 'x is a number'. Similarly, for 'y represents a number', we shall simply say 'y is a number', and so on.

6.2 Basic Operations on Literal Numbers and Numbers

Since the literal numbers are used to represent numbers, all the operations used for two or more numbers in arithmetic are applicable to literal numbers also. Thus, *the literal numbers as well as their combination with numbers obey all the rules (and signs) of addition, subtraction, multiplication and division of numbers along with the properties of these operations.* We now explain this statement.

(i) *Addition*

We know that the sum of two numbers, say 5 and 4, is denoted by $5 + 4$. Similarly, the sum of the literal number x and a number 4 is denoted by $x + 4$. Again, q more than a number p is written as $p + q$; $(a + b) + c$ means that literal number b is to be added to a and then c is to be added to the result.
Also note that $p + q = q + p, p + 0 = p; (p + q) + r = p + (q + r)$

(ii) *Subtraction*

The difference of the literal number x and a number 5 is $x - 5$. More precisely, we may say that 5 less than a number x is $x - 5$. If we have to indicate that b is to be subtracted from a, we write $a-b$. y less than a number x is $x-y$. $(a-b)-c$ means b is to be subtracted from a and then c is to be subtracted from the result.

(iii) *Multiplication*

We know that multiplication is the repeated addition in arithmetic, e.g. $2 + 2 + 2 + 2$ is called 4 times 2 and is written as 4×2. Similarly, $a + a + a + a$ is 4 times a, i.e. $4 \times a$. But the sign of multiplication between a number and a literal number or between two literal numbers is usually omitted because it may be confused with the letter x. Thus, when no

This statement can be abbreviated further by using *letters p and s,* respectively, to represent the perimeter and the length of the side of the square. Hence, if the length of the side is 's' units and the perimeters is 'p' units, then we write

$$p = 4 \times s \qquad \textbf{(1)}$$

This rule is true for all systems of units and for all possible values of the lengths of the side of the square. Note that here p and s represent numbers.

(ii) Let an aeroplane fly at a speed of 1200 km per hour. Then, the distance covered by the aeroplane in 2 hours = 1200 × 2 km = 2400 km. The distance covered by the aeroplane in 3 hours = 1200 × 3 km = 3600 km and the distance covered by the aeroplane in 4 hours = 1200 × 4 km = 4800 km.

From the above, again, we can make a general statement that the distance covered by the aeroplane equals the product of speed and time, i.e.

$$\text{distance} = \text{speed} \times \text{time}$$

This statement can be put briefly further by using *letters s, v and t, respectively, to represent distance, speed and time.* Thus,.

$$s = v \times t \qquad \textbf{(2)}$$

Just like Rule (i), this rule is also true for all systems of units and for all possible values of v and t. Note that here also s, v and t represent numbers.

Thus, from the above two examples, we observe that the use of letters to represent numbers helps us to think in more general terms. In other words, it enables us to solve a problem and obtain a rule (which is generally referred to as a formula) on the basis of which we can solve thousands of different problems of the same type by just changing the values of the letters in the rule (or formula).

The letters which are used to represent numbers are often given the name *literal numbers* or simply *literals.* Note that from now onwards, symbol is used between two literals or between a literal number and a number, it is understood as a multiplication operation between them. Thus,

$$x + x + x + x = 4 \times x = 4x$$

Similarly, *the product of literal numbers x and y is written as xy and both x and y are called the factors of xy.* Also note that $xy = yx$, $x \times 1 = x$, $x \times 0 = 0$, $(xy)z = x(yz)$. Conventionally, the product of the type $a \times 4$ is not written as a4 but as 4a. Similarly, it is also a convention that $1 \times x$ is not written as 1x but simply as x. It may also be noted that in the case of literal numbers $x(y + z) = xy + xz$.

(iv) *Division*

The division sign '÷' read as 'by' between two numbers means that the number on the left of the division sign is to be divided by the number on the right. In the case of literal numbers also $a \div b$ read as 'a by b' means that the literal number a is to be divided by the literal number b and is writen as $\frac{a}{b}$. Thus, 40 divided by p is written as $\frac{40}{p}$ or x divided by y is written as $\frac{x}{y}$.

Let us illustrate these notions through some examples:

Example 1: Anjali scores 100 marks in mathematics and x marks in science. What is her total score in science and mathematics?

Solution: Score in mathematics = 100
Score in science = x
Therefore, total score = 100 + x.

Example 2: Write the following using numbers, literal numbers and signs of basic operations:

(i) 3 more than twice a number x.

(ii) quotient of z by 6 is multiplied by y.

Solution: (i) Twice a number $x = 2 \times x = 2x$

Therefore, 3 more than twice a number $x = 2x + 3$

(ii) Quotient of z by $6 = \frac{z}{6}$. Therefore, quotient of z by 6 is multiplied by y, means $\frac{z}{6} \times y = \frac{zy}{6} = \frac{yz}{6}$

Example 3: Eight times a number p is x less than a number y. Express this statement using literal numbers, numbers and signs of basic operations.

Solution: Eight times a number p means $8 \times p$, i.e. $8p$

x less than a number y means that x is subtracted from y, i.e. $y - x$

Therefore, $8p = y - x$

EXERCISE 6.1

1. Write the following using numbers, literal numbers and signs of basic operations. State what each letter represents?
   (i) The diameter of a circle is twice its radius.
   (ii) The area of a rectangle is the product of its length and breadth.
   (iii) The selling price equals the sum of the cost price and the profit.
2. Write the following using numbers, literal numbers and signs of basic operations:
   (i) The sum of numbers 6 and x
   (ii) 3 more than a number y
   (iii) One-third of a number x
   (iv) One-half of the sum of numbers x and y
   (v) Number y less than a number 7
   (vi) 7 taken away from x
   (vii) 2 less than the quotient of x by y

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6.3 Powers of Literal Numbers

Recall that when a number is multiplied by itself, we write the product in exponential form, e.g. $4 \times 4 = 4^2$, $4 \times 4 \times 4 \times 4 \times 4 = 4^5$. Since a literal number represents a number, this shorthand way of writing the repeated product of a number with itself in the exponential form is applicable to it also. Thus, $a \times a$ is written as a^2 and $a \times a \times a$ is written as a^3, $a \times a \times a \times a \times a$ is written as a^5, and so on.

We read a^2 as the *Second power of a* or *square of a* or *a raised to the exponent* 2 (or simply, *a raised to* 2) or *a squared.* Similarly, a^5 is read as *fifth power of a* or *a raised to exponent* 5 (*or simply, a raised to* 5), and so on.

In a^2, 2 is the *exponent* (or *index*) and a is the *base.* Similarly in a^3, 3 is the *exponent* and a is the *base.* In x^4, 4 is the *exponent* and x is the *base.* The exponent in a power indicate the number of times the base (literal number) has been multiplied by itself. For example,

$y^8 = y \times y \times y \times y \times \ldots$ repeatedly multiplied 8 times
$x^{15} = x \times x \times x \times x \times \ldots$ repeatedly multiplied 15 times

Note that when a^1 appears after simplification or otherwise, we write it as a and not a^1 just as we do not write la but a only. Let us take some examples to illustrate these concepts:

Example 1: Write down the following in product form:
(i) $4a^3$ (ii) x^8 (iii) $7p^2q^3$

Solution: (i) $4a^3 = 4 \times a^3 = 4 \times a \times a \times a$
(ii) $x^8 = x \times x \times x \times x \times x \times x \times x \times x$
(iii) $7p^2q^3 = 7 \times p \times p \times q \times q \times q$

Example 2: Write $6 \times p \times p \times p \times x \times x \times x \times x$ in the exponential form.

Solution: We know that $p \times p \times p = p^3$,
and $x \times x \times x \times x = x^4$
Therefore, $6 \times p \times p \times p \times x \times x \times x \times x = 6p^3x^4$

Example 3: The volume of a cuboid is given by the product of its length, breadth and height. The length of a cuboid is 3 times its breadth and the height is one-half of the length. Find its volume if the breadth is b cm.

Solution: It is given that breadth = b cm

length = 3 × breadth = 3b cm

$$\text{height} = \frac{1}{2}\text{length} = \frac{1}{2} \times 3\text{b cm} = \frac{3}{2}\text{ b cm}$$

Therefore, volume of the cuboid = Length × breadth × height

$$= 3\text{b} \times \text{b} \times \frac{3}{2}\text{ b cm}^3$$

$$= \frac{9}{2}\text{b}^3\text{ cm}^3$$

$$= 4.5\text{ b}^3\text{ cm}^3$$

EXERCISE 6.2

1. Write the following in exponential form:
   (i) a × a × a × ... 10 times
   (ii) 17 × x × x × x × y × y × y
   (iii) y × y × y × y × ... 20 times
   (iv) 7 × a × a × b × b × b × b × b × c
2. Write down the following in product form:
   (i) a^2b^5 (ii) $8z^3$ (iii) $9ab^3$ (iv) $10x^3y^3z^3$
3. The population of a certain species of insects is x now. It becomes y times itself after one week. What will be its population after 2 weeks?

6.4 Variables and Constants

Consider the following examples:

(i) The perimeter of a square of side s is 4s, i.e. $p = 4s$. Here 4 is a fixed number whereas p and s are literal numbers and hence are not fixed, because they depend on the size of the square.

(ii) The distance d in kilometres travelled by light in t seconds is given by $d = 2.99 \times 10^5 t$. Here 2.99×10^5 is a fixed number, but time t and distance d vary because they depend upon time of travel under consideration.

From the above examples, we observe that *a quantity which takes on a fixed numerical value is called a constant and a quantity which takes on various numerical values is called a variable.* In the above examples, 4 and 2.99×10^5 are constants and the literal numbers p, s, d and t are variables. However, it must be noted that in some situations literal numbers are also treated as constants. In such situations, it is presumed that the particular literal number will only take a fixed value.

6.5 Algebraic Expressions

Let us consider the following example to understand the concept of an algebraic expression:

Example 1: Rohini arranged a tour for Class VI students from Delhi to Wardha. They travelled 4 km on foot, 3y km by bus and 2z km by train to complete the journey. Find the total distance travelled by them between Delhi and Wardha.

Solution: Distance travelled on foot = 4 km
Distance travelled by bus = 3y km
and distance travelled by train = 2z km
The total distance covered = $(4 + 3y + 2z)$ km

Here, $4 + 3y + 2z$ is an *algebraic expression* or an *algebraic phrase* which gives the distance in kilometres from Delhi to Wardha. Thus, *a* *number or a combination of numbers (including literal numbers), using the signs of fundamental operation(s), is called an algebraic expression.* For example,

$4.5, \frac{1}{9}, 2x, 4x-9, p+2q-3r, 3a^2+4b^2, a^3+b^3+c^3+3abc, ab+ac,$

$\frac{7x^2}{3}+2$, etc. are all algebraic expressions. $3x$, $-4y$ are the terms of the expression $3x-4y$; $5xy$, $6z^2$, -7 are the terms of the expression $5xy+6z^2-7$.

An expression which contains one term is called a *monomial* (in Greek *mono* means 'single'). $2x$, $5x^2y$, $-6abc$, etc. are examples of monomials. An expression which contains two terms is called a *binomial* (in Latin *bi* means 'double'). $pq+r$, a^2-b^2, $xyz+6$, etc. are all examples of binomials. Similarly, an expression which contains three terms is called a *trinomial* (in Latin *tri* means 'three'). Can you tell what a quadrinomial is? It is an expression which contains four terms like $x^3+y^3+z^3+3xyz$, and so on. Let us take an example to illustrate this classification:

Example 2: Identify the monomials, binomials and trinomials among the following expressions: -7, $x+y$, $4.5a$, a^3-b^3, $a^2+2ab+b^2$, $ax+by+c$, $5xy$.

Solution: Monomials are: -7, $4.5a$, $5xy$
Binomials are: $x+y$, a^3-b^3
Trinomials are: $a^2+2ab+b^2$, $ax+by+c$

A term of the expression which has no literal factor is called a constant term. For example, in $x+2$ and $\frac{4}{3}+a^2+b^2$, the constant terms are 2 and $\frac{4}{3}$, respectively. In the term $3ab$, for instance, 3, a and b are called the *factors* of the term. Clearly, the number 3 is the *numerical* *factor*, and a and b are *literal factors*. Any one of these factors is called the *coefficient* of the product of other factors (with sign) of the term. Thus, the coefficient of b in 3ab is 3a, coefficient of a in 3ab is 3b. Similarly, the coefficient of x in the term $-8xy$ is $-8y$, the coefficient of y is $-8x$ and the coefficient of xy is -8. Sometimes only the numerical factor is referred to as the coefficient of the term. In this sense, we can call -8 as the coefficient of $-8xy$. When the coefficient of a term is $+1$ or -1 the '1' is usually omitted. For instance, we write 1p as p and $-1q$ as $-q$.

Let us take an example:

Example 3: Write down the terms of expression:

$8x^4y - 7x^3yz + \frac{4}{3}x^2yz^2 - 2.5xyz$. What is the coefficient of x^2 in the term $\frac{4}{3}x^2yz^2$?

Solution: The given expression has four terms, namely, $8x^4y$, $-7x^3yz$, $\frac{4}{3}x^2yz^2$, $-2.5xyz$. The coefficient of x^2 in $\frac{4}{3}x^2yz^2$ is $\frac{4}{3}yz^2$.

6.6 Like and Unlike Terms

When the terms have same literal factors they are called *like terms* (or similar terms) otherwise they are called *unlike terms* (or dissimilar terms). For example, in the expression,

$$8x^2y + 6xy^2 - 2xy - 7yx^2$$

$8x^2y$ and $-7yx^2$ are like terms, whereas $6xy^2$ and $-2xy$ are unlike terms. Let us take an example to illustrate these concepts:

Example: In the following which pairs contain like terms?

(i) $3x, -7x$ (ii) $16x, 16y$ (iii) $9ab, -6b$

Solution: (i) We see that the terms $3x$ and $-7x$ have the same literal factors so they are like terms.

(ii) The numerical factors are same but the literal factors are different so they are unlike terms.

(iii) $9ab$ and $-6b$ are unlike terms. (Why?)

6.7 Finding Value of an Algebraic Expression

An algebraic expression involves one or more literal numbers which represent numbers. Thus, to find the value (i.e. numerical value) of an algebraic expression we need to know the numerical values of all the literal numbers appearing in that expression. Then we simply replace the literals by their numerical values to obtain an arithmetic expression and evaluate it by the usual method of arithmetic. The process of replacing the literals by their numerical values is called substitution, i.e. we substitute the numerical values for the literals. Let us take an example to illustrate this:

Example: If $x = 1, y = 2$ and $z = -3$, find the value of

(i) $2xy^4 - 15x^2y + z$

(ii) $x^3 + y^3 + z^3 + 3xyz$

Solution: We substitute $x = 1, y = 2$ and $z = -3$ in each of the expressions. We have:

(i) $$2xy^4 - 15x^2y + z = 2(1)(2)^4 - 15(1)^2(2) + (-3)$$
$$= 2 \times 16 - 15 \times 2 - 3$$
$$= 32 - 30 - 3 = -1$$

(ii) $$x^3 + y^3 + z^3 + 3xyz$$
$$= (1)^3 + (2)^3 + (-3)^3 + 3(1)(2)(-3)$$
$$= 1 + 8 - 27 - 18 = -36$$

EXERCISE 6.3

1. Write all the terms of each of the following algebraic expressions:

(i) $3x^5 + 5y^4 - 7x^2y + 7$

(ii) $9y^3 - 2z^5 + 7x^3y - 3xyz$
(iii) $a^5 - 3ab - b^2 + 6$

2. Identify monomials, binomials and trinomials from the following expressions:
   (i) $x^2 + 2y^2$ (ii) $4x^3$ (iii) $3x^2 + 4y + 5z$
   (iv) $ax^3 + bx + c$ (v) $z + 7$ (vi) $5ab^3$
   (vii) $a^2 + b^2 + c^2 - d^2$ (viii) $4ab$
3. Write down the coefficient of x in the following:
   (i) $3x$ (ii) $-4ax$ (iii) $5xy^2$ (iv) xyz
4. Write the coefficient (numerical) of each term of the expression
   $$x^3 - 7x^2y + 5xy^2 - 2$$
5. Write down the different powers of x in the expression
   $$3x^2 - 2xy + 4y^2x^3.$$
6. Identify like terms in the following:
   (i) $x^2, y^2, 2x^2, z^2$ (ii) $2xy, yz, 3x, \frac{zy}{2}$
   (iii) $-2x^2y,\ x^2z,\ -yx^2,\ x^2y^2$
   (iv) $cab^2,\ a^2bc,\ b^2ac,\ c^2ab,\ ab^2c, abc, acb^2$
7. Evaluate each of the following expressions if $x = 2, y = -3, z = -2, a = 2, b = 3$:
   (i) $2a^2 + 2b$ (ii) $2a^2 + x^2 - y^3$ (iii) $x^3 - y^3 + z^3$
   (iv) $4xy^2 - 3yz^2 + 4x^2z$ (v) $x^3 + y^3 + 3xyz + ab$
   (vi) $5 + 4z^3 - 6y + 7a + xy$

6.8 Operations on Algebraic Expressions

Having stated what algebraic expressions are we need to know how to operate them. Now, we know that an algebraic expression may consist of like terms and unlike terms. The operations of addition and subtraction on algebraic expressions means addition (or subtraction) of like terms. How do we add (or subtract) like terms? Consider, for instance, $4x$ and $7x$. If we have to add them, can we write $4x + 7x = (4 + 7)x$? Yes. Using the property that $x(y + z) = xy + xz$, we simplify the sum.

Thus, $4x + 7x = (4 + 7)x = 11x$. Similarly, the sum of the like terms 3x, 8x and $-5x$ is

$$\begin{aligned} 3x + 8x - 5x &= (3 + 8)x - 5x \\ &= [(3 + 8) - 5]x \\ &= (11 - 5)x = 6x \end{aligned}$$

Therefore, we see that *the sum (or difference) of several like terms is another like term whose coefficient is the sum (or difference) of the coefficients of those like terms.* Let us consider some examples to illustrate:

Example 1: Add $4x^2y$, $+8x^2y$ and $-2x^2y$.

Solution: The sum will be another like term with the coefficient $4 + 8 + (-2) = 10$. Thus, $4x^2y + 8x^2y - 2x^2y = 10x^2y$. Alternatively, we can use the distributive property of multiplication over addition and write $4x^2y + 8x^2y - 2x^2y = (4 + 8 - 2)x^2y = 10x^2y$.

Example 2: Subtract 30xy from 12xy.

Solution: $12xy - 30xy = (12 - 30)xy = -18xy$

In adding or subtracting algebraic expressions, we collect different groups of like terms and find the sum or difference of like terms in each group by the rule stated above. The collection can be done by adopting the following two methods:

(i) *Horizontal method:* All expressions are written in a horizontal line (row) and then the terms are arranged to collect all the groups of like terms together and then added or subtracted.

(ii) *Column method:* In this case each expression is written in a separate row such that their like terms are arranged one below the other in a column. Then addition or subtraction of the terms is done columnwise.

Let us take some examples to illustrate these methods:

Example 3: Add the expressions $3x^2 + 4y - 5z^3$, $5y + 2x^2$, $7x^2 - 8y$ and $4x^2 - 9y - 5z^3$.

Solution: *Column method*: We write the expressions so that their like terms are in a column as shown below (For this purpose, the order of the terms in the expressions can be changed):

$$\begin{array}{l}
3x^2 + 4y - 5z^3 \\
2x^2 + 5y \qquad \text{(Order has been changed)} \\
7x^2 - 8y \\
4x^2 - 9y - 5z^3 \\
\hline
16x^2 - 8y - 10z^3 \\
\hline
\end{array}$$

Horizontal method:

$$\begin{aligned}
& 3x^2 + 4y - 5z^3 + (5y + 2x^2) + (7x^2 - 8y) + \\
& (4x^2 - 9y - 5z^3) \\
= {} & 3x^2 + 2x^2 + 7x^2 + 4x^2 + 4y + 5y - 8y - 9y \\
& - 5z^3 - 5z^3 \\
= {} & (3 + 2 + 7 + 4)x^2 + (4 + 5 - 8 - 9)y + \\
& (-5 - 5)z^3 \\
= {} & 16x^2 - 8y - 10z^3
\end{aligned}$$

Example 4: From $(4x + 3y)$ subtract $(2x - 4y)$.

Solution: We have

$$\begin{aligned}
& (4x + 3y) - (2x - 4y) \\
= {} & 4x + 3y - 2x - (-4y) \\
= {} & 4x + 3y - 2x + 4y \\
= {} & (4 - 2)x + (3 + 4)y = 2x + 7y
\end{aligned}$$

Recall that the negative of a negative number is positive, i.e. $2 - (-4) = (2 + 4) = 6$ and the negative of a positive number is negative, i.e. $2 - (4) = 2 - 4 = -2$. So we can say that *to subtract an expression from another, we should change the sign (from '+' to '—' or from '—' to '+') of each term in the expression which is to be subtracted and then add the two expressions.* In the subtraction of two expressions, in the column method, we indicate the change of sign of every term in the expression to be subtracted below the original sign of each term.

For example, we can subtract $2x - 4y$ from $4x + 3y$ as shown below:

$$\begin{array}{rcr}
4x & + & 3y \\
2x & - & 4y \\
\underline{-} & \underline{+} & \\
\hline
2x & + & 7y \\
\hline
\end{array}$$

Example 5: From the sum of $3x + 2y + 3z$ and $3x - 4y + 5z$ subtract $6x + 7y - 2z$.

Solution: We write the expression in separate rows so that their like terms are in a column with the change of the sign in the last row of the expression (which is to be subtracted). We have:

$$\begin{array}{rrrr}
 & 3x & + 2y & + 3z \\
 & 3x & - 4y & + 5z \\
 & 6x & + 7y & - 2z \\
 & - & - & + \\
\hline
+ & 0x & - 9y & + 10z \\
\hline
\end{array}$$

Thus, we have $0x - 9y + 10z = -9y + 10z$

Example 6: From the sum of $4x^4 - 3x^3 + 6x^2$, $4x^3 + 4x - 3$ and $-3x^4 - 5x^2 + 2x$ subtract $5x^4 - 7x^3 - 3x + 4$.

Solution: We write the expressions row-wise with like terms arranged in a column and change the signs of all terms in the expression to be subtracted before performing the addition of the coefficients columnwise. We have:

$$\begin{array}{rrrrr}
4x^4 & - 3x^3 & + 6x^2 & & \\
 & + 4x^3 & & + 4x & - 3 \\
- 3x^4 & & - 5x^2 & + 2x & \\
5x^4 & - 7x^3 & & - 3x & + 4 \\
- & + & & + & - \\
\hline
- 4x^4 & + 8x^3 & + x^2 & + 9x & - 7 \\
\hline
\end{array}$$

EXERCISE 6.4

1. Add the following:

   (i) $x^3, -3x^3, 2x^3, -4x^3$

   (ii) $2x^2y, -4x^2y, 6x^2y, -5x^2y$

   (iii) $2xyz, -5xyz, 7xyz$

2. Simplify the following:

   (i) $23a^5 - 15a^5$ (ii) $-15ab - 3ab$

   (iii) $12x^2 - 15x^2$

   (iv) $-3x^2y - 5x^2y$ (v) $-x^2 + 3x^2 - 7x^2 + 12x^2$

   (vi) $2z^2b - 5z^2b - 6a^2b - 3a^2b$

3. Add the following expressions:

   (i) $a + 2b + 3c$ and $a + b - c$

   (ii) $x + 2y + z, x + y + 2z$ and $x - y - z$

   (iii) $x^2 + y^2$ and $2x^2 + 3y^2 + 3$

   (iv) $2xy - yz - zx, 2yz - zx - yx$ and $2zx - xy - zy$

   (v) $a^2 + 2ab + b^2, a^2 - 2ab + b^2$ and $a^2 - b^2$

4. Add the following:

   (i) $$\begin{array}{r} 2x^2 + 4y^2 + 6 \\ -x^2 - 5y^2 - 8 \\ -3x^2 - 2y^2 + 4 \end{array}$$

   (ii) $$\begin{array}{rrr} 4xy & - 2yz & + 7zx \\ -3xy & + 5yz & - 8zx \\ -6xy & & - 2zx \end{array}$$

5. Subtract:

   (i) $5a - 3b$ from $2a + b - 2$

   (ii) $-x^2 - 3z$ from $5x^2 - y + z + 7$

   (iii) $-2a + b + 4d$ from $4a - 2b - c$

6. Subtract $-11r^2s^2 + 7rs - 6$ from $9r^2s^2 - 4rs + 8$.

7. What should be added to $a^2 + 2ab + b^2$ to obtain $4ab + b^2$?

8. Subtract the sum of $13x - 4y + 7z$ and $-6z + 6x + 3y$ from the sum of $6x - 4y, -4z + 2x$ and $4y - 7$.

9. From the sum of $a^2 + 4b^2 - 6ab, a^2 - b^2 + 2ab, b^2 + 6$ and $a^2 - 4ab$ subtract $-2a^2 + b^2 - ab + a$.

6.9 The Use of Grouping Symbols

At times, it is necessary to indicate that an expression consisting of two or more terms is to be considered as a single number. The symbols for groupings, namely *parentheses* ( ), *brackets* [ ] or *braces* { }, are used to set apart such expressions. For example $2h \times l + b)$ means that l is to be added to b and this sum is to be multiplied by 2h.

Thus, we need to insert or remove the grouping symbols while performing (algebraic) operations on expressions to simplify the result. Recall that while adding two expressions, e.g.

$$(a^2 - 4ab + 7) + (-4a^2 + 3ab - 8)$$

we first remove the parentheses from each expression and write

$$a^2 - 4ab + 7 - 4a^2 + 3ab - 8$$

This means that if a plus sign precedes a grouping symbol, we remove the parentheses and write the expression as it is without any change.

Similarly, while subtracting an expression from another expression, e.g. in $(a^2 + b^2 - 2ab) - (a^3 - b^3 + 3ab)$, we remove the parentheses and write the first expression as it is. (Why?) What sign precedes the parentheses in the second expression? Minus sign. This, of course, means that we have to subtract the second expression from the first one. We, therefore, remove the parentheses and change the sign of each term in the expression. Thus, we obtain the following:

$$a^2 + b^2 - 2ab - a^3 + b^3 - 3ab$$

So we state the following rules for the removal of grouping symbols:

(i) *If a '+' sign precedes a symbol of grouping, the grouping symbol may be removed without any change in the sign of the terms.*

(ii) *If a '−' sign precedes a symbol of grouping, the grouping symbol may be removed and the sign of each term is changed.*

(iii) *If more than one grouping symbol is present in an expression, we remove the innermost grouping symbol first and collect and combine like terms, if any. We continue this process outwards until all the grouping symbols have been removed.*

Let us illustrate these rules through some examples:

Example 1: Simplify $4x^3 - [9x^2 - \{-5x^3 - (2 - 7x^2) + 6x\}]$.

Solution: Remove the innermost grouping symbol ( ) first, then { } and then bracket [ ]. Thus, we have:

$$= 4x^3 - [9x^2 - \{-5x^3 - (2-7x^2) + 6x\}]$$
$$= 4x^3 - [9x^2 - \{-5x^3 - 2 + 7x^2 + 6x\}]$$
$$= 4x^3 - [9x^2 + 5x^3 + 2 - 7x^2 - 6x]$$
$$= 4x^3 - [(9-7)x^2 + 5x^3 + 2 - 6x]$$
$$= 4x^3 - [2x^2 + 5x^3 + 2 - 6x]$$
$$= 4x^3 - 2x^2 - 5x^3 - 2 + 6x$$
$$= 4x^3 - 5x^3 - 2x^2 + 6x - 2$$
$$= -x^3 - 2x^2 + 6x - 2$$

Example 2: Simplify and find the value of the following expression when $a = 3$ and $b = 1$:

$4(a^2 + b^2 + 2ab) - [4(a^2 + b^2 - 2ab) - \{-b^3 + 4(a - 3)\}]$

Solution: We work outwards from the innermost symbol. Thus,

$$4(a^2 + b^2 + 2ab) - [4(a^2 + b^2 - 2ab) - \{-b^3 + 4(a-3)\}]$$
$$= 4a^2 + 4b^2 + 8ab - [4a^2 + 4b^2 - 8ab - \{-b^3 + 4a - 12\}]$$
$$= 4a^2 + 4b^2 + 8ab - [4a^2 + 4b^2 - 8ab + b^3 - 4a + 12]$$
$$= 4a^2 + 4b^2 + 8ab - 4a^2 - 4b^2 + 8ab - b^3 + 4a - 12$$

$= 4a^2 - 4a^2 + 4b^2 - 4b^2 + 8ab + 8ab - b^3 + 4a - 12$

$= (4-4)a^2 + (4-4)b^2 + (8 + 8)ab - b^3 + 4a - 12$

$= 16ab - b^3 + 4a - 12$

The value of the simplified expression (for $a = 3$ and $b = 1$) is $16 \times 3 \times 1 - (1)^3 + 4 \times 3 - 12$

$= 48 - 1 + 12 - 12 = 47$

Now, how do we insert the grouping symbols around expressions? We use similar rules as (i) and (ii) discussed earlier. *If a grouping symbol preceded by a '+' sign is to be inserted, we simply insert the* ***symbol around the expression.*** In this case, we do not disturb the signs of the terms in the expression. However, if a ***grouping symbol preceded by a '—' sign is to be inserted, we change the sign of each term in the expression and insert the symbol preceded by the '—' sign.*** Let us take an example to illustrate this rule.

Example: Place the last two terms of the following expression in parentheses preceded by a minus sign:

$x + y - 3z + 7$

Solution: We have to insert parentheses around the last two terms preceded by a minus sign. We have:

$x + y - 3z + 7 = x + y - (3z - 7)$

EXERCISE 6.5

1. Simplify the following:
   (i) $(a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab)$
   (ii) $-5(a + b) + 2(2a - b) + 4a - 7$
   (iii) $-x^2 + [-(3x^2 + 2y^2) + (x^2 - 4)]$
   (iv) $3x^2z - 4yz + 3xy - [x^2z - (x^2z - 3yz) - 4yz - 7z]$
   (v) $-m - [m + \{m + n - 2m - (m - 2n)\} - n]$

(vi) $a - \frac{1}{3}[(a^2 - 5b) - 2\{2a^2 - (3c - 2b)\}]$

(vii) $85 - [12x - 7(8x - 3) - 2\{10x - 5(2 - 4x)\}]$

(viii) $15x - [8x^3 + 3x^2 - \{8x^2 - (4 - 2x - x^3) - 5x^3\} - 2x]$

(ix) $xy - [yz - zx - \{yx - (3y - xz) - (xy - zy)\}]$

(x) $5 + [x - \{2y - (6x + y - 4) + 2x^2\} - (x^2 - 2y)]$

2. Place the last two terms in each of the following expressions in parentheses preceded by a '−' sign:
    (i) $9a + 5xy - 7x^2 + 8y - 6$
    (ii) $-y + z + x^2 - y^2 - a^2$
    (iii) $x + y + z - xy - yz - zx$
    (iv) $xy^2 + yz^2 + zx^2$

Things to Remember

1. The letters which are used to represent numbers are called literal numbers or literals.
2. The literal numbers themselves as well as the combinations of literal numbers and numbers obey all the rules (and signs) of addition, subtraction, multiplication and division of numbers along with the properties of these operations.
3. $x \times y = xy, 4 \times x = 4x, 1 \times x = x, x \times 7 = 7x$
4. $y \times y \times y \times \ldots 8 \text{ times} = y^8, a \times a \times a \times \ldots 6 \text{ times} = a^6$. Generally, in x^2, p^3, y^8, a^6, etc., 2, 3, 8, 6 are called indices or exponents and x, p, y and a are bases.
5. A quantity which takes a fixed numerical value is called a constant.

6. A quantity which takes on various numerical values is called a variable.
7. A number or a combination of numbers (including literal numbers), using the signs of fundamental operations, is called an algebraic expression.
8. Two or more signs ('+' or '−') separate an expression into several parts. Each part along with its sign is called a term of the expression.
9. An expression which contains one term is called a monomial, which contains two terms is called a binomial and which contains three terms is called a trinomial.
10. The terms having same literal factors are called like terms, otherwise they are called unlike terms.
11. The sum (or difference) of several like terms is another like term whose coefficient is the sum (or difference) of those like terms.
12. In adding or subtracting algebraic expressions, we collect different groups of like terms and find the sum or difference of like terms in each group.
13. To subtract an expression from another, we can change the sign (from '+' to '−' and from '−' to '+') of each term of the expression to be subtracted and then add the two expressions.
14. When a grouping symbol preceded by a '−' sign is removed or inserted, then the sign of each term of the corresponding expression is changed (from '+' to '−' and from '−' to '+').

CHAPTER SEVEN

Introduction of Linear Equations

IN THIS CHAPTER, we shall study the meaning of an equation, in general, and of a linear equation, in particular. We shall find the solution of a linear equation in one variable by trial-and-error and also by a systematic method involving addition, subtraction, multiplication or division of the same number on both sides of the equation. We shall also formulate equations for some simple real-life problems and solve them.

7.1 Linear Equations

We have already come across statements of the following type in earlier chapters:

$$3 + 2 = 5 \qquad \textbf{(1)}$$
$$4 \times (5 + 6) = 4 \times 5 + 4 \times 6 \qquad \textbf{(2)}$$
$$a(b + c) = ab + ac \qquad \textbf{(3)}$$

Let us now consider the following statements:

(i) x more than a number 4 is 9.
(ii) 7 less than a number x is 6.
(iii) 9 times a number x is 12.
(iv) A number y divided by 6 gives 2.
(v) y multiplied by itself is 5 more than it.
(vi) The sum of number x and twice the number z is 15.
(vii) The third power of m is 27.
(viii) 3 less from twice a number p is 15.

Clearly, we can write the above statements (i) to (viii), respectively, as follows:

$$4 + x = 9 \quad \textbf{(4)}$$

$$x - 7 = 6 \quad \textbf{(5)}$$

$$9 \times x = 12 \quad \textbf{(6)}$$

$$y \div 6 = 2 \quad \textbf{(7)}$$

$$y^2 = y + 5 \quad \textbf{(8)}$$

$$x + 2z = 15 \quad \textbf{(9)}$$

$$m^3 = 27 \quad \textbf{(10)}$$

$$2p - 3 = 15 \quad \textbf{(11)}$$

We observe that in each of the statements (1) to (11), the symbol '=' (is equal to) appears. A statement involving symbol '=' is called a statement of equality or simply an *equality*. Thus, each of the statement (1) to (11) is an equality. We note that statements (1) and (2) do not involve any literal number (variable) while all the remaining statements (3) to (11) involve one or more than one literal number. *A statement of equality which involves one or more literal numbers is called an equation*. Thus, each of the equalities (3) to (11) is an equation. Note that an equation has two sides, namely, the left hand side (written as L.H.S.) and the right hand side (written as R.H.S.). Thus, in equation (3), $a(b + c)$ is L.H.S. and $ab + ac$ is R.H.S. while in equation (7), $y \div 6$ is L.H.S. and 2 is R.H.S. We further observe that equation (3) is true for all values of a, b and c, i.e. we can assign any values to a, b and c and still the L.H.S. will be equal to the R.H.S.

However, in the case of equations (4) to (11) we observe that these are not true for all values of x, y, etc. The literal numbers involved in each equation are called its variables (*unknowns*). (Usually the variables are denoted by letters from the latter part of the English alphabet, e.g. x, y, z, u, v, r, etc.).

We can easily see that equation (9) is in two variables while each of the equations (4), (5), (6), (7), (8), (10) and (11) are in only one variable. (Why?) Further, in each of the equations (4), (5), (6), (7), (9) and (11) the highest powers of the variables involved is one. An equation in which the highest powers of the variables involved is one is called a *linear equation*. Thus, each of the equations (4), (5), (6), (7), (9) and (11) is a linear equation. In this chapter, we shall limit our discussion to linear equations in one variable only.

Let us take an example to illustrate how we can translate a real-life situation into the form of an equation:

Example: A motor-boat uses 0.1 litre of fuel for every kilometre. One day it made a trip of x km. Form an equation in x if the total consumption of fuel was 10 litres.

Solution: Fuel used in 1 km trip = 0.1 litre
Fuel used in x km trip = $0.1 \times x$ litres
Therefore, total fuel used = $0.1x$ litres
But it is given as 10 litres.
Hence, $0.1x = 10$ is the required equation in x.

EXERCISE 7.1

1. Write the L.H.S. and the R.H.S. of the following equations:
   (i) $x - 2 = 2$ (ii) $2y = 9 - y$
   (iii) $2p = 6$ (iv) $2x + y = 7 + z$
2. Translate each of the following statements into an equation, using y as the variable:
   (i) 3 less than twice a number is 17.
   (ii) Twice of a number subtracted from 15 is 7.
   (iii) A number multiplied by itself is 6 more than the number.
   (iv) One-sixth of a number is 7.
   (v) Twice of a number added to 6 gives 30.
   (vi) Twice of a number when divided by 3 gives 10.

7.2 Solution of an Equation

Let us consider the linear equation in one variable, namely,

$$x - 8 = -4 \quad \textbf{(1)}$$

L.H.S. of (1) is $x - 8$ and its R.H.S. is -4.

Now we evaluate the L.H.S. of (1) for some values of x and continue to give new values till the L.H.S. becomes equal to the R.H.S.

x	*L.H.S.*	*R.H.S.*
0	−8	−4
1	−7	−4
2	−6	−4
3	−5	−4
4	−4	−4

We observe that the L.H.S. equals the R.H.S. only when 4 is substituted for x. For all other values of x the equation is not true because the L.H.S. is not equal to the R.H.S.

A number, which when substituted for the variable in the equation, makes its L.H.S. equal to the R.H.S., is said to *satisfy* the equation and is called a *solution* or a *root* of the equation. Finding the root (s) of an equation is called *solving* the equation. In the above method, we often make a guess of the solution of the equation. This is often referred to as the trial-and-error method. Let us take one more example to illustrate this method:

Example 1: Find the solution of the equation $z - 1 = -3 + 2z$ by the trial-and-error method.

Solution: We try several values of z and find the L.H.S. and the R.H.S. We stop when for a particular value of z the L.H.S. is equal to the R.H.S.

z	*L.H.S.*	*R.H.S.*
0	−1	−3
1	0	−1
−1	−2	−5
−2	−3	−9
2	1	1

Therefore, $z = 2$ is the solution of the given equation.

EXERCISE 7.2

1. Solve the following equations by the trial-and-error method:

(i) $x + 7 = 12$ (ii) $x - 15 = 20$

(iii) $5x = 30$ (iv) $14 - x = 8$

(v) $z - 2 = -6$ (vi) $19 = 7 + x$

(vii) $\frac{x}{8} = 9$ (viii) $3x + 4 = 5x - 4$

(ix) $\frac{1}{3}x + 8 = 11$

7.3 Solving an Equation

We have learnt above the method of solving an equation by trial-and-error. We find that it takes time and is not always direct. Therefore, let us try to have a better method of solving an equation.

An equation can be compared with a balance used for weighing. Its sides are two pans and the equality symbol '=' tells us that the two pans are in balance (Fig. 7.1).

You must have seen the working of a balance. If we put equal weights in both the pans, then we observe that the two pans remain in balance. Similarly, if we remove equal weights from both the pans, we observe that the pans still remain in balance. Thus, we can add (and hence multiply) equal weights or amounts to both pans or we can subtract (and hence divide) equal amounts from both pans and the balance will remain undisturbed. Similarly, in the case of an equation we can

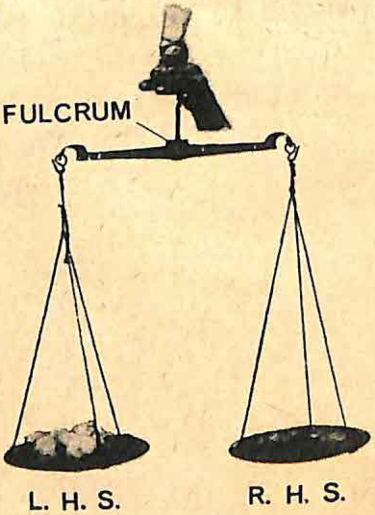


Fig. 7.1: Weighing Balance

(i) *add the same number to both sides of the equation, i.e.*

$$\text{if } x + 2 = 5, \text{ then } x + 2 + 3 = 5 + 3$$

(ii) *subtract the same number from both sides of the equation, i.e.* if $x + 2 = 6$, then $x + 2 - 1 = 6 - 1$

(iii) *multiply both sides of the equation by the same number, i.e.*

$$\text{if } \frac{x}{6} = 5, \text{ then } \frac{x}{6} \times 12 = 5 \times 12$$

(iv) *divide both sides of the equation by the same (non-zero) number, i.e.*

$$\text{if } 5x = 12, \text{ then } 5x \div 5 = 12 \div 5.$$

Now, we shall solve some equations using the above rules:

Example 1: Solve $7 + x = 5$

Solution: Subtracting 7 from both sides of the equation (Rule ii), we get

$7 + x - 7 = 5 - 7$

or, $x + 7 - 7 = -2$

or, $x + 0 = -2$ (Using $a + 0 = a$)

or, $x = -2$

So $x = -2$ is the solution of the given equation.

Check: Let us verify by substituting $x = -2$ in the given equation.

L.H.S. $= 7 - 2 = 5$

R.H.S. $= 5$

i.e. for $x = -2$, L.H.S. = R.H.S.

Example 2: Solve the equation $y - 5 = 7$.

Solution: We add 5 to both sides of the given equation (Rule i). We get

$y - 5 + 5 = 7 + 5$

or, $y + 0 = 12$

or, $y = 12$ (Using $a + 0 = a$)

Thus, $y = 12$ is the solution of the given equation.

Check: Let us substitute $y = 12$ in the given equation. We get

L.H.S. $= 12 - 5 = 7$

R.H.S. $= 7$

i.e. for $y = 12$, L.H.S. = R.H.S.

Example 3 : Solve the equation $\frac{y}{12} = 48$.

Solution : We multiply both sides of the equation by 12 (Rule iii). We get

$$\frac{y}{12} \times 12 = 48 \times 12$$

or, $y = 576$

Thus, $y = 576$ is the solution of the given equation.

Check : Let us put $y = 576$ in the given equation. We get

L.H.S. $= \frac{576}{12} = 48$

R.H.S. $= 48$

i.e. for $y = 576$, L.H.S. = R.H.S.

Example 4 : Solve the equation $15x = 21$.

Solution : We divide both sides of the equation by 15 (Rule iv). We get

$$\frac{15x}{15} = \frac{21}{15}$$

or, $x = \frac{7}{5}$

Thus, $x = \frac{7}{5}$ is the solution of the given equation.

Check : Let us put $x = \frac{7}{5}$ in the given equation. We get

L.H.S. $= 15 \times \frac{7}{5} = 21$

R.H.S. $= 21$

i.e. for $x = \frac{7}{5}$, L.H.S. = R.H.S.

Example 5: Solve the equation $11x + 2 = -20$.

Solution: We subtract 2 from both sides of the equation (Rule ii). We get

$11x + 2 - 2 = -20 - 2$

or, $11x + 0 = -22$

or, $11x = -22$ (Using $a + 0 = a$) **(1)**

Now, we divide both sides of the equation (1) by 11 (Rule iv). We get

$11x \div 11 = -22 \div 11$

or, $x = -2$

Thus, $x = -2$ is the required solution.

Check: Let us put $x = -2$ in the given equation. We get

L.H.S. $= 11(-2) + 2 = -22 + 2 = -20$

R.H.S. $= -20$

i.e. for $x = -2$, L.H.S. = R.H.S.

Example 6: Solve the equation $2x - \frac{1}{2} = 3$.

Solution: We add $\frac{1}{2}$ to both sides of the equation (Rule i). We get

$$2x - \frac{1}{2} + \frac{1}{2} = 3 + \frac{1}{2}$$

or, $2x = \frac{7}{2}$

Now, we divide both sides of the equation by 2 (Rule i). We get

$$2x \div 2 = \frac{7}{2} \div 2$$

or, $x = \frac{7}{4}$

Thus, $x = \frac{7}{4}$ is the required solution.

Check: Let us put $x = \frac{7}{4}$ in the given equation. We get

$$\text{L.H.S.} = 2 \times \frac{7}{4} - \frac{1}{2} = \frac{7}{2} - \frac{1}{2} = \frac{6}{2} = 3$$

$$\text{R.H.S.} = 3$$

i.e. for $x = \frac{7}{4}$, L.H.S. = R.H.S.

We observe that in solving an equation, we use one or more of the above four rules and attempt to end up at a step in which the *variable appears by itself as one side of the equation.*

EXERCISE 7.3

Solve each of the following equations and check your answers:

1. $x + 9 = -3$
2. $y - \frac{11}{2} = 16$
3. $2y = 7$
4. $15x = 225$
5. $\frac{x}{5} = 15$
6. $-7x = 14$
7. $2y + 5 = 8$
8. $9z - 7 = 14$
9. $10 - z = 6$
10. $z - \frac{1}{2} = 3$
11. $7 + 4y = -5$
12. $17u = 255$
13. $\frac{x}{7} = 25$
14. $y + \frac{3}{2} = 5$
15. $3(x + 6) = 21$
16. $5(x + 3) = 15$

(*Hint:* In the last two questions, first remove the grouping symbol.)

7.4 Application of Equations to Practical Problems

Some practical problems involve relations among unknown and known numbers, which can be expressed using mathematical quantities. Thus, we get equations for these problems. These relations are often stated in words and it is for this reason that we often refer to these problems as word problems. We have already learnt to formulate equations for some of the problems of these types. The important steps in the solution of such word problems are:

(i) Read the problem carefully and note down what is given and what is required.
(ii) Denote the unknown by some letter, say x, y, z, etc.
(iii) Translate the statements of the problem step/word by step/word into mathematical statements, to the extent it is possible.
(iv) Look for the quantities which are equal and write an equation by writing appropriate expressions for these quantities.
(v) Solve the equation for the unknown.
(vi) Check whether the solution satisfies the conditions given in the problem.

Let us consider some examples to illustrate these steps:

Example 1 : The sum of the ages of father and his son is 75 years. If the age of the son is 25 years, find the age of the father.

Solution : Let the age of the father be x years.

The age of son = 25 years

Therefore, the sum of the two ages = $(x + 25)$ years.

But this sum is given as 75 years. Hence, we have the following equation:

$$x + 25 = 75$$

Subtracting 25 from both sides of this equation, we have

$x + 25 - 25 = 75 - 25$

or, $x = 50$

Thus, the age of the father is 50 years.

Check : Sum of the two ages = (50 + 25) = 75 years, which is the same as given.

Example 2 : There are only 25-paise coins in a purse. If the total value of the money in the purse is Rs 15, find the number of coins in it.

Solution : Let the number of coins be x.

Value of one coin = 25 paise

Therefore, value of the total coins = $25 \times x$ paise

$$= \text{Rs } \frac{25x}{100} = \text{Rs } \frac{x}{4}$$

But this value is given as Rs 15.

Hence, we have the following equation:

$$\frac{x}{4} = 15$$

Multiplying both sides of the above equation by 4, we have

$$\frac{x}{4} \times 4 = 15 \times 4$$

$$\text{or, } x = 60$$

Thus, the number of coins in the purse is 60.

Check : Value of coins = 60×25 paise = Rs $\frac{60 \times 25}{100}$

= Rs 15, which is the same as given.

Example 3 : The length of a rectangle is three times its width. If the perimeter of the rectangle is 96 metres, find the length and the breadth of the rectangle.

Solution : Let the width of the rectangle be x metres.

Its length = 3 times the width

$= 3 \times x = 3x$ metres

Therefore, its perimeter = $2(l + b)$

$= 2(3x + x)$ metres = $8x$ metres

But it is given as 96 metres.

Therefore, $8x = 96$

or, $\frac{8x}{8} = \frac{96}{8}$

or, $x = 12$

Thus, the width of the rectangle = 12 metres.

Hence, its length = 3 × 12 metres = 36 metres.

Check: Perimeter = 2 (36 + 12) metres = 96 metres, which is the same as given.

EXERCISE 7.4

1. 5 added to a number gives 9. Find the number.
2. What is the number which when multiplied by 20 gives the product 60?
3. Find the number which when divided by 9 gives 4.
4. Rahim's father is three times as old as Rahim. If sum of their ages is 56 years, find their ages.
5. Rita has 18 metres of cloth. She wants to cut it in two pieces in such a way that one piece will be 4 metres longer than the other. What will be the length of the shorter piece?
6. The total number of students in a school is 1260. If the number of girls is 52 more than that of the boys, find the number of boys in the school. Also, find the number of girls.
7. There are only 10-paise coins in a purse. If the total value of the money in the purse is Rs 25, find the number of coins in the purse.
8. After 15 years, Salma will be four times as old as she is now. Determine her present age?
9. In a factory, the number of woman employees is three times that of the man employees. If the total number of employees in the factory is 48, find the number of man employees in the factory. Also, find the number of woman employees.
10. Find two numbers such that one is five times the other and their difference is 20.

11. Anjali is 4 years older than Babita. If the sum of their ages is 16, find their ages.
12. Lakshmi Devi has a rectangular plot which has been fenced with a 300 m long wire. Find the length and the breadth of the plot if the length is twice its width.
13. The sum of two consecutive numbers is 53. Find the numbers.
14. The sum of two consecutive even numbers is 86. Find the numbers.
15. In a family, the consumption of wheat is three times that of rice. If the total consumption of the two cereals in the family in a particular month was 36 kg, find the consumption of rice in that particular month. Also, find the consumption of wheat.
16. The difference between the years of birth and death (in A.D.) of famous Egyptian woman mathematician Hypatia is 40 years. Two times the year of her birth exceed the year of her death by 335 years. Find the year of her birth.

Things to Remember

1. A statement of equality involving one or more literal numbers is called an equation.
2. An equation involving only one literal number with the highest power one is called a linear equation in one variable.
3. While solving an equation we can
   - (i) add the same number to both sides of the equation,
   - (ii) subtract the same number from both sides of the equation,
   - (iii) multiply both sides of the equation by the same number, and
   - (iv) divide both sides of the equation by the same (non-zero) number.

UNIT THREE

COMMERCIAL MATHEMATICS

COMMERCIAL MATHEMATICS includes ratio and proportion, percentage, profit and loss, interest, taxation and other commercial or business activities. The history of commercial activities is as old as human civilization. The primitive people used to hunt and produce according to their needs for a day or two. But, with the development of civilization, they learnt to live in a community and started growing more than the need. The surplus food or things were bartered for other commodities of daily use, which promoted the idea of commerce and trade. Subsequently, 'money' also came into use to make this exchange easier and thus trade flourished. From the clay tablets of the period 2450 B.C. to 2330 B.C., it is evident that the Babylonians were familiar with bills, promissory notes, mortgages, taxes, simple and compound interests and other commercial activities.

The Indian mathematicians Brahmagupta and Bhaskara are well known for giving the idea of the Rule of Three which was highly regarded by merchants as a tool for centuries. Brahmagupta (around A.D. 628) stated the rule as follows:

> "In the Rule of Three, Argument, Fruit and Requisition are the names of the terms. The first and the last terms must be similar. The Requisition multiplied by the Fruit and divided by the Argument is the Produce."

Another Indian mathematician Mahavira (around A.D. 850) also stated this rule in nearly the same form as follows:

> "*Phala* (फल) multiplied by *Ichcha* (इच्छा) and divided by *Pramana* (प्रमाण) becomes the answer, when the *Ichcha* and *Pramana* are similar."

It is only at the end of the fourteenth century, it was recognized that the Rule of Three has some connection with the idea of proportion. The symbol ':' to denote a ratio and the symbol to denote the equality of two ratios appear to have been introduced by an English mathematician Oughtred in A.D. 1628.

The Romans made use of fractions which easily reduce to hundredths without recognizing per cents as such. In the Italian manuscripts of commercial mathematics of the fifteenth century, it is common to find expressions such as '20 p 100', 'x p cento', and 'vi p c^{o}' for 20%, 10% and 6% respectively. Thus, the per cent sign was initially written as 'per c^{o}', 'pc^{o}', etc. In the middle of the seventeenth century, it had developed into the form 'per $\frac{o}{o}$'.

Finally, 'per' was dropped and it took the present form '%'. The Indian mathematician Bhaskara has used per cents in the interest problems in his famous work *Lilavati*. In the sixteenth century, per cents were mainly used for the computation of interest and profit and loss.

The phrase 'profit and loss' had been in use in the same sense as we use it today. The popularity of this topic in the sixteenth century can be inferred from the fact that a mathematician Werner has devoted forty-seven pages to it in his book *Rechenbuch* (around A.D. 1561). The Unitary Method was also well known during this period which is evident from the solution provided by an Italian mathematician Tartagalia (around A.D. 1556) for the problem:

> If 1 pound of silk costs 9 lire 18 soldi, how much will 8 ounces cost?

Here, in Chapter Eight, we shall discuss some basic concepts related to ratio, proportion and unitary method. In Chapter Nine, the concept of percentage will be introduced and it will be applied to profit and loss, simple interest and other daily life problems.

CHAPTER EIGHT

Ratio, Proportion and Unitary Method

IN THIS CHAPTER, we shall learn some basic concepts of ratio and proportion. These concepts have been applied to solve some real-life problems. Unitary method has been explained and some problems based on direct variation only have been solved using this method.

8.1 Ratio

The weight of a bag of wheat is 100 kg and the weight of a bag of fertilizer is 40 kg. How do their weights compare? We can compare the weights in two different ways:

(i) The difference of the two weights is $(100 - 40)$ kg or 60 kg. We say that the weight of wheat is 60 kg more than that of fertilizer. This is known as *comparison by difference.*

(ii) For the above, we can also say that the weight of wheat is $(100 \div 40)$, i.e. $\left(\frac{100}{40}\right)$ times the weight of fertilizer. This is known as *comparison by division.*

When we compare two quantities of the same kind (with respect to magnitudes) by division, we say that we have formed a *ratio* of the two quantities. Thus, we can say that the ratio of the weight of wheat to that of fertilizer is $100 \div 40$ or $\frac{100}{40}$. Usually, the symbol ':' is used to express a ratio. Therefore, the above ratio is written as 100 : 40 and read as '100 is to 40' or '100 to 40'. Similarly, if the cost of a tractor is thirty times the cost of a pair of bullocks, we can write that the ratio of the cost of a tractor to that of a pair of bullocks is 30 : 1. In general, the ratio of two numbers (measures/magnitudes) a and b ($b \neq 0$) is $a \div b$ (or $\frac{a}{b}$) and is denoted as a : b. a and b are called the *terms* of the ratio. For obvious reasons, a is the first term and b is the second term of the ratio a : b. The first term of a ratio is called the *antecedent* and the second term is called the *consequent.* Thus, in the ratio a : b, a is the antecedent and b is the consequent.

Since a quotient does not change when the dividend and the divisor are multiplied or divided by the same number, say c ($c \neq 0$), one and the same ratio can be expressed in several ways. For example, 100 : 40 = 10 : 4 = 5 : 2 = 125 : 50, etc. Note that in the ratio 5 : 2, the two terms 5 and 2 have *no common factor other than 1.* The ratio expressed in this form is said to be in the *simplest form* or in the *lowest terms.* Usually, *a ratio is expressed in the simplest form.*

It may be carefully noted that the comparison becomes meaningless if the quantities being compared are not of the same kind (i.e. of the same units of length, volume, or currency, etc.). It is just meaningless to compare 10 bags with 200 cows. Therefore, to find the ratio of two quantities, they must be expressed in the same units. It may also be noted that the order of the terms in a ratio is very important. It can be easily seen that the ratio of 6 to 11 is different from the ratio of 11 to 6, i.e. 6 : 11 is different from 11 : 6. Let us now take some examples to illustrate these ideas :

Example 1: The number of boys and girls in a school are 480 and 384, respectively. Express the ratio of the number of boys to that of the girls in the simplest form.

Solution: The required ratio is $480 : 384 = \frac{480}{384}$.

To express it in the simplest form, we need to find the H.C.F. of 480 and 384. It is 96. Then we divide each term of the ratio by the H.C.F., i.e. 96. We get

$$\frac{480 \div 96}{384 \div 96} = \frac{5}{4} \text{ or } 5 : 4$$

Thus, the ratio of the number of boys to that of the girls is 5 : 4.

Example 2: The length of a steel tape for measurements of buildings is 10 m and its width is 2.4 cm. What is the ratio of its length to the width?

Solution: Ratio of length to width $= \dfrac{10 \text{ m}}{2.4 \text{ cm}}$

$$= \frac{10 \times 100 \text{ cm}}{2.4 \text{ cm}} \text{ (since 1 m = 100 cm)}$$

$$= \frac{10000}{24} = \frac{1250}{3}$$

Thus, the required ratio is 1250 : 3.

Example 3: An office opens at 9 a.m. and closes at 5 p.m. with a lunch interval of 30 minutes. What is the ratio of lunch interval to the total period in office?

Solution: Note that we can write 9 a.m. as 9 hours and 5 p.m. as 17 hours. Total period in office = (17 − 9) hours

= 8 hours

Lunch interval = 30 minutes.

Therefore, the required ratio $= \dfrac{30 \text{ minutes}}{8 \text{ hours}}$

$$= \frac{30 \text{ minutes}}{8 \times 60 \text{ minutes}} = \frac{1}{16} \text{ or } 1:16$$

Example 4: Find the ratio of 250 g to 5 kg. Also, find the ratio of 5 kg to 250 g.

Solution: Ratio of 250 g to 5 kg $= \dfrac{250 \text{ g}}{5 \text{ kg}}$

$$= \frac{250 \text{ g}}{5 \times 1000 \text{ g}} \qquad (1 \text{ kg} = 1000 \text{ g})$$

$$= \frac{1}{20} = 1:20$$

$$\text{Ratio of 5 kg to 250 g} = \frac{5\text{ kg}}{250\text{ g}}$$

$$= \frac{5 \times 1000\text{ g}}{250\text{ g}}$$

$$= \frac{20}{1} = 20:1$$

From the above examples, we can easily observe that a ratio has no units in itself.

EXERCISE 8.1

1. Express the following as ratios:
   - (i) In a class the number of girls in the merit list of the board examination is six times that of boys.
   - (ii) One part of oxygen and two parts of hydrogen by volume combine to form water.
   - (iii) The number of students passing the mathematics test is two-third of the number that appeared.
2. Express each of the following ratios in its simplest form:
   - (i) 150 : 400 (ii) 85 : 255
   - (iii) a dozen to a score (iv) 27 : 57
3. Find the ratio of the following in the simplest form:
   - (i) 35 minutes to 45 seconds
   - (ii) 90 paise to Rs 3
   - (iii) 8 kg to 400 g
   - (iv) 1 hour to 15 seconds
4. Sunita earned Rs 40000 and paid Rs 5000 as income tax. Find the ratio of (i) income tax to income and (ii) income to income tax.

5. Avinash works as a lecturer and earns Rs 2000 per month. His wife who is a doctor earns Rs 2500 per month. Find the following ratios:
   (i) Avinash's income to the income of his wife.
   (ii) Avinash's income to their total income.

6. Margarette works in a factory and earns Rs 955 per month. She saves Rs 185 per month from her earnings. Find the ratio of
   (i) her savings to her income.
   (ii) her income to her expenditure.
   (iii) her savings to her expenditure.

7. Find the ratio of
   (i) 3.2 metres to 56 metres
   (ii) 10 metres to 25 cm
   (iii) 25 paise to Rs 60
   (iv) 270 to 450
   (v) 10 litres to 0.25 litres
   (vi) 200 kg to 40 kg

8. Of the 72 persons working in an office, 28 are men and the remaining are women. Find the ratio of the number of
   (i) men to that of women.
   (ii) men to the total number of persons.
   (iii) persons to that of women.

9. A bullock-cart travels 24 km in 3 hours and a train travels 120 km in 2 hours. Find the ratio of their speeds.

$$\left[\textit{Hint}:\quad \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right]$$

10. In India, the number of villages electrified in 1951 is about 3000 and the number of villages electrified in 1984 is about 350000. Find the ratio of the number of villages electrified in the two years.

8.2 Proportion

Consider the following examples:

(i) If milk costs Rs 5 per litre, the cost of 40 litres of milk is Rs 200 and the cost of 70 litres is Rs 350. The ratio of the two quantities of milk is $40:70 = 4:7$ and the ratio of their cost is $200:350 = 4:7$. Hence, $40:70 = 200:350$.

(ii) 200 litres and 150 litres of kerosene oil weigh 180 kg and 135 kg, respectively. The ratio of the two volumes is $200 : 150 = 4 : 3$ and the ratio of the two weights is $180:135 = 4:3$. Thus, $200:150 = 180:135$.

We see that in these examples, the ratio of the first two numbers is equal to the ratio of the third and the fourth numbers. *Such an equality of two ratios is called a proportion* and we say that the four numbers are *in proportion.* For instance, in the first example, the numbers 40, 70, 200 and 350 are in proportion and in the second example the numbers 200, 150, 180 and 135 are in proportion.

(iii) A train moving at 40 km per hour will take 5 hours to travel a distance of 200 km. Another train moving at 50 km per hour will take 4 hours to cover the same distance. Then, the ratio of the speeds $= 40 : 50 = 4 : 5$ and the ratio of the time taken $= 50:40 = 5:4$. Thus, $40:50 \neq 50:40$.

In such a case we say that the numbers 40, 50, 50 and 40 *are not in proportion.* In general, the numbers a, b, c, d are in proportion if the ratio of the first two is equal to the ratio of the last two, i.e.

$$a:b = c:d$$

This is sometimes expressed by the notation

$$a:b::c:d$$

Which is read as 'a is to b as c is to d' or 'a to b as c to d'. a, b, c and d are the first, second, third and fourth terms of the proportion. The first and the fourth terms of a proportion are called *extreme terms or extremes.* The second and the third terms are called the *middle terms or means* (see Fig. 8.1).

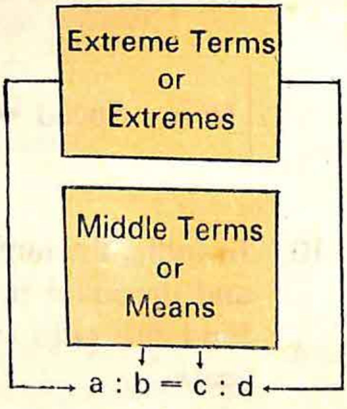


Fig. 8.1

In Example (i) the numbers involved are: 40, 70, 200, 350. Thus, the first and the fourth terms are 40 and 350 and the middle terms are 70 and 200, respectively. Further,

Product of extreme terms = $40 \times 350 = 14000$

Product of middle terms = $70 \times 200 = 14000$

We see that the product of the extreme terms is equal to the product of the middle terms.

Similarly in Example (ii), the product of the extreme terms 200×135 is equal to the product of the middle terms 150×180. But in Example (iii) the product of the extreme terms 40×40 *is not equal to* the product of the middle terms 50×50.

Thus, we observe that if the four numbers are in proportion, then the product of the extreme terms is equal to the product of the middle terms. In other words,

if $a : b = c : d$

then, $ad = bc$

Clearly, if $a : b \neq c : d$, then $ad \neq bc$. To understand these concepts, let us consider some examples:

Example 1: Are 40, 30, 60, 45 in proportion?

Solution: We have $40 : 30 = \frac{40}{30} = \frac{4}{3}$

and $60 : 45 = \frac{60}{45} = \frac{4}{3}$

Thus, $40 : 30 = 60 : 45$

i.e. 40, 30, 60, 45 are in proportion.

Example 2: The first, third and fourth terms of a proportion are 12, 8 and 14, respectively. Find its second term.

Solution: Let the second term be x.

Thus, 12, x, 8, 14 are in proportion.

Now the product of the extreme terms = $12 \times 14 = 168$

and the product of the middle terms = $x \times 8 = 8x$

Hence, $8x = 168$

or, $\frac{8x}{8} = \frac{168}{8}$

or, $x = 21$

Thus, the second term of the proportion is 21.

Example 3: Observe that $100 \times 75 = 150 \times 50$. Check if the numbers 100, 150, 50, 75 are in proportion.

Solution: $100 : 150 = \frac{100}{150} = \frac{2}{3}$

and $50 : 75 = \frac{50}{75} = \frac{2}{3}$

Clearly, $100 : 150 = 50 : 75$

Hence, the numbers 100, 150, 50, 75 are in proportion.

Example 4: Observe that $100 \times 150 \neq 50 \times 75$. Check if the numbers 100, 50, 75, 150 are in proportion.

Solution: $100 : 50 = \frac{100}{50} = \frac{2}{1}$

and $75 : 150 = \frac{75}{150} = \frac{1}{2}$

Clearly, $100 : 50 \neq 75 : 150$

Hence, the numbers 100, 50, 75, 150 are not in proportion.

By taking several such products as in Examples 3 and 4 above, we can easily observe that *if ad = bc, then a, b, c, d are in proportion and if ad ≠ bc, then a, b, c, d are not in proportion.* This is an important property and it helps us to check quickly whether the four given numbers are in proportion or not.

Example 5: Are the numbers 5, 6, 20, 18 in proportion?

Solution: $5 \times 18 = 90$

and $6 \times 20 = 120$

Clearly, $5 \times 18 \neq 6 \times 20$

Thus, 5, 6, 20, 18 are not in proportion.

Example 6: Are the numbers 3, 9, 9, 27 in proportion?

Solution: $3 \times 27 = 81$

and $9 \times 9 = 81$

Clearly, $3 \times 27 = 9 \times 9$

Thus, 3, 9, 9, 27 are in proportion.

In the above proportion, we observe that the middle term 9 is *repeated.* In such a case, we say that *the numbers 3, 9, 27 are in proportion.* In other words, *if* $a : b = b : c$, *then* a, b, c *are in proportion.* In this case, $b^2 = ac$ and the middle term b is called the *mean proportional* between a and c.

Example 7: Verify that

(i) $30 : 45 = 16 : 24$

(ii) $30 : 16 = 45 : 24$

(iii) $16 : 30 = 24 : 45$

(iv) $45 : 30 = 24 : 16$

Solution: (i) $30 : 45 = \frac{30}{45} = \frac{2}{3}$

and $16 : 24 = \frac{16}{24} = \frac{2}{3}$

Thus, $30 : 45 = 16 : 24$

(ii) $30 : 16 = \frac{30}{16} = \frac{15}{8}$

and $45 : 24 = \frac{45}{24} = \frac{15}{8}$

Thus, $30 : 16 = 45 : 24$

(iii) $16 : 30 = \frac{16}{30} = \frac{8}{15}$

and $24 : 45 = \frac{24}{45} = \frac{8}{15}$

Thus, $16 : 30 = 24 : 45$

(iv) $45 : 30 = \frac{45}{30} = \frac{3}{2}$

and $24 : 16 = \frac{24}{16} = \frac{3}{2}$

Thus, $45 : 30 = 24 : 16$

From the above examples, we observe that from a given proportion, we can form *three more proportions* by just changing the positions of the terms of the proportion.

Example 8: Set up all possible proportions from the numbers 15, 18, 35 and 42.

Solution: We note that $15 \times 42 = 630$ and $18 \times 35 = 630$.

Thus, $15 \times 42 = 18 \times 35$ **(1)**

Hence, $15 : 18 = 35 : 42$

(1) can be written as $15 \times 42 = 35 \times 18$

Hence, $15 : 35 = 18 : 42$

(1) can also be written as $42 \times 15 = 18 \times 35$

Hence, $42 : 18 = 35 : 15$

Lastly, (1) can also be written as $42 \times 15 = 35 \times 18$

Hence, $42 : 35 = 18 : 15$

Thus, the required proportions are:

(i) $15 : 18 = 35 : 42$
(ii) $15 : 35 = 18 : 42$
(iii) $42 : 18 = 35 : 15$
(iv) $42 : 35 = 18 : 15$

Example 9: The ratio of the number of girls to that of boys participating in sports in a school is 5 : 4. If the number of girls is 210, determine the number of boys participating in the sports.

Solution: Let the number of boys be x. Therefore, the ratio of the number of girls to that of boys $= 210 : x$

But it is given as $5 : 4$

Therefore, $5 : 4 = 210 : x$

week of a grocery shop was 2 : 9. If the total sale of eggs in the same week was Rs 360, find the sale of eggs on the Sunday.

9. In a school library, the ratio of mathematics books to science books is the same as the ratio of science books to Hindi books. If there are 450 books in science and 300 books in Hindi, find the number of books in mathematics.
10. The ratio of copper and zinc in an alloy is 9 : 7. If the weight of zinc in the alloy is 9.8 kg, find the weight of copper in the alloy.
11. Find x if 25, 35, x are in proportion.
12. The ratio of the income to the expenditure of a family is 7 : 6. Find the savings if the income is Rs 1400.
    [*Hint:* First find the expenditure.]
13. The scale of a map is, usually, indicated in one corner of the map by writing the ratio of the distance on the map to the corresponding distance on the ground. This ratio is called the Representative Fraction (R.F.) of the map. The scale of a map is 1:100000. What actual distance would 4 cm on the map represent?
    [*Hint:*The scale tells us that 1 cm on the map represents 100000 cm actual distance. Let actual distance be x cm. Then 1 : 100000 = 4 : x. Recall 100 cm = 1 m, 1000 m = 1 km.]

8.3 Unitary Method

Let us consider the following example:

Example 1: The cost of 3 notebooks is Rs 16.50. What will be the cost of 7 notebooks?

Solution: From the given information, we first find the cost of 1 notebook and then calculate the cost of 7 notebooks.

Thus, we have:

Cost of 3 notebooks = Rs 16.50

Therefore, the cost of 1 notebook = Rs (16.50 ÷ 3) = Rs 5.50 (Since the cost of 1 notebook will be less than the cost of 3 notebooks.)

$$\text{Hence, the cost of 7 notebooks} = \text{Rs } 5.50 \times 7$$
$$= \text{Rs } 38.50$$

Thus, the cost of 7 notebooks is Rs 38.50.

Note that here we have first found the cost of 1 notebook and then calculated the cost of the required number of notebooks. In other words, we have first found the value of one (unit) quantity from the value of the given quantities and then found the value of the required number of quantities. This method of finding the values is called the *Unitary Method.* Let us take some more examples to illustrate this method:

Example 2: An aeroplane flies 4000 km in 5 hours. How far does it travel in 3 hours?

Solution: The distance travelled is related to the time of flight. If the time of flight increases the distance travelled also increases. Here the unknown quantity is the *distance travelled* and the known quantity is *time.* So we first work out the value of the unknown quantity (distance travelled) for unit value of known quantity (1 hour) as follows:

$$\text{Distance travelled in 5 hours} = 4000 \text{ km}$$

$$\text{Therefore, the distance travelled in 1 hour} = \frac{4000}{5} \text{ km}$$
$$= 800 \text{ km}$$

$$\text{Hence, the distance travelled in 3 hours} = 800 \times 3 \text{ km}$$
$$= 2400 \text{ km}$$

Thus, the aeroplane travels 2400 km in 3 hours.

Example 3: A woman worker earns Rs 18000 in 15 months.

(i) How much does she earn in 7 months?

(ii) In how many months will she earn Rs 30000?

Solution: We note that the income is related to the period of work. In fact if the period of work is more, the income is more and their ratio remains the same.

(i) *The income is the unknown* and the *period of* *work is the known* quantity. Hence, we proceed as follows:

Income for 15 months = Rs 18000

Therefore, income for 1 month = Rs $\frac{18000}{15}$

= Rs 1200

Hence, income for 7 months = Rs 1200 × 7

= Rs 8400

Thus, she earns Rs 8400 in 7 months.

(ii) *The number of months is the unknown* quantity and *the income is the known* quantity. So, we have:

Number of months required to earn Rs 18000 = 15

Therefore, number of months required to earn Re 1 = $\frac{15}{18000}$

Hence, number of months required to earn

$$\text{Rs } 30000 = \frac{15}{18000} \times 30000 = 25$$

Thus, she will earn Rs 30000 in 25 months.

In all the above examples, we see that two quantities are related in some way. If the value of one quantity increases (or decreases), the value of the other quantity also increases (or decreases) in such a manner that the ratio of the values of the two quantities remains the same. We say that the two quantities are in *direct variation.* In such cases, if the value of one quantity is multiplied (or divided) by some number, the value of the other quantity is also multiplied (or divided) by the same number. However, we do have situations in which two quantities are so related that if the value of one quantity increases (or decreases), the value of the other quantity decreases (or increases) in such a manner that the product of the two values remains the same. For example, consider the following situation:

'10 workers can do a piece of a work in 18 days. In how many days will 36 workers do the same work?'

Obviously, in this situation when the number of workers increases, the number of days decreases and when the number of workers decreases, the number of days increases in such a manner that the product of the two values always remains 10×18, i.e. 180. Such problems are also solved by unitary method. These types of problems will be discussed in higher classes.

EXERCISE 8.3

1. The cost of 30 metres of polyester cloth is Rs 450. Find the cost of 16 metres of cloth?
2. A worker is paid Rs 162.50 for 5 days. What should be paid to him for 28 days?
3. The monthly consumption of cereals of a hostel with 400 students is 5200 kg. Find the consumption if the number of students is only 65.
4. A car travels 165 km in 3 hours.
   (i) How long will it take to travel 440 km?
   (ii) How far will it travel in $6\frac{1}{2}$ hours?
5. The weight of 45 folding chairs is 18 kg. How many chairs can be loaded on a truck having a capacity of carrying 4000 kg load?
6. If 6 oil tankers can be filled by a pipe in $4\frac{1}{2}$ hours, how long does the pipe take to fill 4 such oil tankers?
7. The yield of wheat from 6 hectares is 280 quintals. Find the number of hectares required for a yield of 225 quintals.
8. The cost of 17 chairs is Rs 9605. Find the number of chairs that can be purchased in Rs 56500.
9. The weight of 72 books is 9 kg.
   (i) What is the weight of 80 such books?

(ii) How many such books weigh 6 kg?

10. The price of 3 metres of cloth is Rs 79.50. Find the price of 15 metres of such cloth.
11. Fifteen postcards cost Rs 2.25. What will be the cost of 36 postcard? How many postcards can we buy in Rs 45?

Things to Remember

1. The ratio of a number a to another number b ($b \neq 0$) is the quotient $a \div b$ and is written as $a : b$ where a and b are called its terms.
2. The ratio of two numbers is usually expressed in its simplest form.
3. An equality of two ratios is called a proportion. The numbers a, b, c, d are in proportion if the ratio of the first two is equal to the ratio of the last two.
4. The first and the fourth terms of a proportion are called the extremes or extreme terms. The second and the third terms are called the means or middle terms.
5. Four numbers are in proportion if the product of extreme terms is equal to the product of middle terms.
6. From the terms of a given proportion, we can make three more proportions.
7. If $a : b = b : c$, then a, b, c are said to be in proportion.
8. If the middle terms are repeated then each of the middle term is called the mean proportional.
9. The method of finding first the value of one (unit) quantity from the value of the given quantities and then the value of the required quantities is called the Unitary Method.

CHAPTER NINE

Percentages and Their Applications

IN THIS CHAPTER, we shall study the concept of percentage. We shall learn to convert a per cent into a fraction, ratio or decimal and vice versa. The concept of percentage shall be applied to profit and loss, simple interest and other real-life problems.

9.1 Per cent

In many areas of everyday life, we come across with the concept of percentage. For example, we hear the following type of statements:

(i) Anita scored 93 per cent marks in mathematics.
(ii) Fans are sold at 5 per cent discount in the off season.
(iii) The percentage of people living in villages is 70.
(iv) The bank pays interest at the rate of 5 per cent.
(v) The pass percentage of girls was 87 and those of boys was 76 in an examination, and so on.

In all the above statements, a comparison has been made. Let us now consider the following example:

Suppose in School A, 256 students passed out of 320 students and in School B, 300 students passed out of 400 students. Can we say that the performance of School B was better because 300 > 256? No. If we want to compare the performances of the two schools, we shall have to compare $\frac{256}{320}$ with $\frac{300}{400}$ (or 256: 320 with 300 : 400) instead of comparing 256 with 300. Can we now say which is better of the two? No.

Let us look at the two fractions (or the two ratios) in their simplest form, i.e. $\frac{4}{5}$ and $\frac{3}{4}$ (or 4 : 5 and 3 : 4). Even then we cannot tell which school has a better performance. Note that the two fractions (or the two ratios) can be compared if their denominators (or the second terms), respectively, are the same.

Let us, therefore, rewrite the above fractions with the same denominator, say 100 (or the two ratios with the second term 100). We have:

$$\textit{School A}: \ \frac{4}{5} = \frac{4}{5} \times \frac{100}{100} = \frac{4}{5} \times 100 \times \frac{1}{100} = \frac{80}{100}$$

(or $4 : 5 = 80 : 100$)

$$\textit{School B}: \ \frac{3}{4} = \frac{3}{4} \times \frac{100}{100} = \frac{3}{4} \times 100 \times \frac{1}{100} = \frac{75}{100}$$

(or $3 : 4 = 75 : 100$)

It is now obvious that School A has better performance than School B.

A fraction with its denominator as 100 is called a per cent. Similarly, a ratio with its second term 100 is called a per cent. The per cent is an abbreviation of the Latin phrase *per centum*, meaning per hundred or hundredths. For convenience, the symbol '%' is used for 'per cent'. The per cent stands for multiplication with $\frac{1}{100}$. For example, the above performances are written as

$\frac{80}{100} = 80 \times \frac{1}{100} = 80$ hundredths $= 80$ per hundred
$= 80$ per cent $= 80\%$

$\frac{75}{100} = 75 \times \frac{1}{100} = 75$ hundredths $= 75$ per hundred
$= 75$ per cent $= 75\%$

Similarly, $80 : 100 = 80\%$ and $75 : 100 = 75\%$. The statement '15 per cent increase' means an increase of 15 per hundred, i.e. an increase of 15 for every hundred. Let us now take some examples to illustrate these concepts:

Example 1: Express the following as per cents:

(i) $\frac{2}{3}$ (ii) 30 : 80 (iii) 6 : 5 (iv) 0.065

Solution: (i) $\frac{2}{3} = \frac{2}{3} \times \frac{100}{100} = \frac{2}{3} \times 100 \times \frac{1}{100} = \frac{2}{3} \times 100\%$

$= 66\frac{2}{3}\%$

(ii) $30 : 80 = \frac{30}{80} = \frac{30}{80} \times \frac{100}{100}$

$= \frac{30}{80} \times 100\%$

$= 37\frac{1}{2}\%$

We see that to convert a ratio into a per cent, we write it as a fraction and multiply it by 100.

(iii) $6 : 5 = \frac{6}{5} \times 100\% = 120\%$

(iv) $0.065 = \frac{65}{1000} = \frac{65}{1000} \times 100\%$

$= 6.5\%$

We see that to convert a decimal into a per cent, we shift the decimal point by two places to the right (since it is multiplied by 100).

Example 2: Express the following per cents as fractions and ratios in the simplest forms. Express them as decimals also.

(i) 52% (ii) 115%

Solution (i): As a fraction: $52\% = 52 \times \frac{1}{100} = \frac{13}{25}$

As a ratio: $52\% = 52 : 100 = 13 : 25$

As a decimal : $52\% = 52 \times \frac{1}{100} = 0.52$

(ii) $115\% = 115 \times \frac{1}{100} = \frac{23}{20}$

$115\% = 115 \div 100 = \frac{115}{100} = 23 : 20$

$115\% = 115 \times \frac{1}{100} = 1.15$

Thus, we observe that to convert a per cent:

(i) *into a fraction, we drop the per cent sign (%) and divide the remaining number by 100.*

(ii) *into a ratio, we drop the per cent sign (%) and form a ratio with the remaining number as the first term and 100 as the second term.*

(iii) *into a decimal, we again drop the per cent sign (%) and shift the decimal point two places to the left.*

Example 3: Find 30% of Rs 180.

Solution: To find certain per cent of a given quantity, we multiply the given per cent with the given quantity.

Thus, 30% of Rs 180 $= \frac{30}{100} \times$ Rs 180

$= \text{Rs}\ \frac{30}{100} \times 180 = \text{Rs}\ 54$

Hence, 30% of Rs 180 is Rs 54.

Example 4: What per cent of 25 kg is 3.5 kg?

Solution: The required per cent $= \frac{3.5\ \text{kg}}{25\ \text{kg}} \times 100$

$$= \frac{3.5 \times 100}{25} = \frac{35 \times 100}{250}$$

$$= \frac{7 \times 100}{50} = \frac{14 \times 100}{100} = 14$$

Example 5: Rahim obtained 60 marks out of 75 in mathematics. Find the percentage of marks obtained by Rahim in mathematics.

Solution: The required per cent $= \dfrac{60 \times 100}{75} = \dfrac{20 \times 100}{25}$

$$= \frac{80 \times 100}{100} = 80$$

Thus, the percentage of marks obtained by Rahim in mathematics is 80.

Example 6: In an orchard, $16\frac{2}{3}$% of the trees are apple trees. If the number of trees in the orchard is 240, find the number of other types of trees in the orchard.

Solution: Total number of trees = 240

The number of apple trees $= 16\frac{2}{3}\%$ of 240

$$= \frac{50}{3}\% \text{ of } 240$$

$$= \frac{50}{3} \times \frac{1}{100} \text{ of } 240$$

$$= \frac{50}{300} \times 240 = 40$$

Therefore, the number of other types of trees

$= 240 - 40 = 200$

Thus, the number of other types of trees in the orchard is 200.

Example 7: The approximate population of India was 5480 lakh in 1971 and was 6850 lakh in 1981. Find the per cent increase in the population of India from 1971 to 1981.

Solution: Population in 1981 = 6850 lakh

Population in 1971 = 5480 lakh

Therefore, increase in population = (6850 − 5480) lakh
= 1370 lakh

Now, increase in population on 5480 lakh = 1370 lakh

Therefore, increase on 1 lakh $= \frac{1370}{5480}$ lakh

Hence, increase on 100 lakh $= \frac{1370}{5480} \times 100$ lakh

= 25 lakh

i.e. increase on 100 = 25

Thus, increase per cent = 25

Alternatively: We can solve this question as follows:

$$\text{Increase per cent} = \frac{1370 \text{ lakh}}{5480 \text{ lakh}} \times 100 = \frac{1370}{5480} \times 100$$

$$= \frac{1}{4} \times 100 = \frac{25}{100} \times 100 = 25$$

Note: Increase (or decrease) per cent is calculated by the ratio (or fraction) formed by the increase (or decrease) and the *original value.* In the above case, population of 1971 is the original value.

EXERCISE 9.1

1. Express the following as ratios in the simplest form:

   (i) 18% (ii) $12\frac{1}{2}$% (iii) 140% (iv) $6\frac{1}{4}$%

   (v) 2% (vi) 0.3%

2. Express the following as fractions in the simplest form:

   (i) $16\frac{2}{3}$% (ii) 45% (iii) 150% (iv) 0.25%

EXERCISE 4.4

1. Determine the H.C.F. of numbers in each of the following by prime factorization method:
   - (i) 144, 198
   - (ii) 81, 117
   - (iii) 47, 61
   - (iv) 225, 450
   - (v) 13,39,273
   - (vi) 150, 140, 210
   - (vii) 120, 144, 204
   - (viii) 106, 159, 265
   - (ix) 101, 573, 1079
   - (x) 625, 3125, 15625
2. Determine the H.C.F. of numbers in each of the following by continued division method:
   - (i) 300, 450
   - (ii) 442, 1261
   - (iii) 252, 576
   - (iv) 935, 1320
   - (v) 1624, 522, 1276
   - (vi) 2241, 8217, 747
3. It is given that 65610 is divisible by 27. Which two numbers nearest to 65610 are each divisible by 27?
4. What is the H.C.F. of any two consecutive numbers?
5. Reduce each of the following fractions to the lowest terms by cancelling the H.C.F. of the numerator and the denominator:

   (i) $\frac{1444}{256}$ (ii) $\frac{143}{234}$ (iii) $\frac{2211}{5025}$
6. Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.
7. Two tankers contain 850 litres and 680 litres of petrol, respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times.
8. Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11 and 15, respectively.
9. The length, breadth and height of a room are 8m 25 cm, 6 m 75 cm and 4m 50cm, respectively. Determine the longest tape which can measure the three dimensions of the room exactly.
10. A rectangular courtyard is 20 m 16 cm long and 15 m 60 cm broad. It is to be paved with square stones of the same size. Find the least possible number of such stones.

4.8 Lowest Common Multiple (L.C.M.)

We recall that the lowest common multiple (L.C.M.) of two or more numbers is the smallest number which is a multiple of each of the given numbers. Let us find the L.C.M. of 8 and 12.

Multiples of 8 : 8, 16, 24, 32, 40, 48, . . .
Multiples of 12 : 12, 24, 36, 48, 60, 72, . . .

Common multiples of 8 and 12: 24, 48, . . .
The smallest of the common multiples of 8 and 12: 24

To find the L.C.M. of two or more numbers by prime factorization method, *we write prime factorization of each of the numbers. We then find the product of all the different prime factors of the numbers using each common prime factor the greatest number of times it appears in the prime factorization of any of the numbers.*

Example 13: Find the L.C.M. of 40, 36 and 126.

Solution: We write prime factorization of each number and have

$$40 = 2 \times 2 \times 2 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3$$
$$126 = 2 \times 3 \times 3 \times 7$$

We note that 2 occurs as a prime factor maximum three times, 3 two times, 5 one time and 7 one time. Therefore, required L.C.M. $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$

$= 2520$

We also use division method to find the L.C.M. of the given numbers. In this process, *we set aside all common factors of two or more given numbers by division. The product of all the common factors set aside and of those that remain is the required L.C.M. of the given numbers.*

Example 14: Determine the L.C.M. of 112, 140 and 168.

Solution: We use division method to find the L.C.M. of 112, 140 and 168 and have

2	112	140	168
2	56	70	84
2	28	35	42
7	14	35	21
	2	5	3

Therefore, L.C.M. $= 2 \times 2 \times 2 \times 7 \times 2 \times 5 \times 3$

$= 1680$

Example 15: Determine the lowest natural number which when divided by 16, 28, 40, 77 leaves remainder 8 in each case.

Solution: We know that the lowest number divisible by 16, 28, 40, 77 is their L.C.M. Therefore, the required number must be 8 more than their L.C.M. Now to find their L.C.M. we have

2	16	28	40	77
2	8	14	20	77
2	4	7	10	77
7	2	7	5	77
	2	1	5	11

Therefore, L.C.M. $= 2 \times 2 \times 2 \times 7 \times 2 \times 5 \times 11$

$= 6160$

Hence, the required number $= 6160 + 8$

$= 6168$

4.9 Some Properties of H.C.F. and L.C.M.

In view of our discussions in this chapter, we here list certain facts concerning the H.C.F. and the L.C.M. of numbers.

1. The H.C.F. of given numbers is not greater than any of the numbers.
2. The L.C.M. of given numbers is not less than any of the numbers.
3. H.C.F. of two coprime numbers is 1. Therefore, to determine whether or not two numbers are coprime we see whether or not their H.C.F. is 1.
4. Obviously, the L.C.M. of two or more coprime numbers is their product.
5. If a number, say a, is a factor of another number, say b, then the H.C.F. of a and b is a and their L.C.M. is b.
6. Since the H.C.F. of two or more numbers exactly divides each of the given numbers and each of the given numbers exactly divides their L.C.M., therefore, their H.C.F. must be a factor of their L.C.M. Consequently, there cannot exist two numbers whose H.C.F. is not a factor of their L.C.M.

We now discover a very important property concerning the H.C.F. and the L.C.M. of two numbers. Let us take the numbers 35 and 40.

H.C.F. of 35 and 40 : 5
L.C.M. of 35 and 40 : 280
Product of H.C.F. and L.C.M. = 5×280
= 1400
Product of the given numbers = 35×70
= 1400

Let us consider one more pair of numbers, say 50 and 60.

H.C.F. of 50 and 60 : 10
L.C.M. of 50 and 60 : 300
Product of H.C.F. and L.C.M. = 3000
Product of the numbers = 50×60
= 3000

In each case we find that *the product of the H.C.F. and the L.C.M. of two numbers is equal to the product of the given numbers.* The proof of this property is beyond the scope of the present textbook. However, this can be verified by taking some other examples of numbers.

This property is sometimes used to find the L.C.M. of two very large numbers as we get

$$\text{L.C.M. of two numbers} = \frac{\text{Product of the numbers}}{\text{H.C.F. of the numbers}}$$

Example 16: On a particular day, from Delhi to Meerut buses ran at intervals of 40 minutes while from Meerut to Delhi they ran at intervals of 45 minutes. At what earliest time would two buses coming from opposite directions pass a particular bridge if they passed the same bridge at 10.15 a.m.?

Solution: The earliest time, in minutes, should be the L.C.M. of 40 and 45 added to 10.15 a.m.

L.C.M. of 40 and 45 = 360

Therefore, the two buses would have passed the same bridge after 360 minutes, i.e. 6 hours after 10.15 a.m. Hence, the two buses would pass the bridge at 4.15 p.m.

EXERCISE 4.5

1. Determine the L.C.M. of numbers in each of the following:
   (i) 48, 60 (ii) 18, 77
   (iii) 12, 15, 45 (iv) 15, 30, 90
   (v) 45, 105, 165 (vi) 6, 15, 18, 30
   (vii) 180, 384, 144 (viii) 240, 420, 660
   (ix) 108, 135, 162 (x) 112, 168, 266
2. For each of the following pairs of numbers show that the product of their H.C.F. and L.C.M. equals their product:
   (i) 14, 21 (ii) 117, 221
   (iii) 25, 65 (iv) 27, 90

3. Determine the lowest common denominator for fractions in each of the following:

   (i) $\frac{3}{65}, \frac{17}{91}, \frac{5}{117}$ (ii) $2\frac{4}{15}, \frac{7}{135}, \frac{8}{105}$

4. Given that the H.C.F. of two numbers is 16 and their product is 6400, determine their L.C.M.
5. Is the product of three numbers always equal to the product of their H.C.F. and L.C.M.?
6. The H.C.F. and the L.C.M. of two numbers are 13 and 1989, respectively. If one of the numbers is 117, determine the other.
7. Can two numbers have 14 as their H.C.F. and 204 as their L.C.M. Give reasons in support of your answer.
8. In a school there are two sections – Section A and Section B – of Class VI. There are 32 students in Section A and 36 in Section B. Determine the minimum number of books required for their class library so that they can be distributed equally among students of Section A or of Section B.
9. Telegraph poles occur at equal distances of 220 m along a road and heaps of stones are put at equal distances of 300 m along the same road. The first heap is at the foot of the first pole. How far from it along the road is the next heap which lies at the foot of a pole?
10. Find the smallest number which when divided by 25, 40 and 60 leaves remainder 7 in each case.
11. A boy saves Rs 4.65 daily. Find the least number of days in which he will be able to save an exact number of rupees.
12. In a morning walk three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm, respectively. At what distance from the starting point will they again step off together?
13. Determine the two numbers nearest to 10000 which are exactly divisible by each of 2, 3, 4, 5, 6 and 7.
14. Determine the number nearest to 100000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21.

Things to Remember

1. A factor of a number divides the number exactly.
2. A multiple of a number is exactly divisible by the number.
3. Every number is a factor as well as a multiple of itself.
4. 1 is a factor of every number and is the only number, which is neither prime nor composite.
5. 2 is the only even prime number.
6. Prime factorization of a number is unique irrespective of the order of its factors.
7. Primes occurring in pairs with a difference of two are called twin primes.
8. The product of H.C.F. and L.C.M. of two numbers equals their product.
9. Coprime numbers have 1 as their only common factor. Thus the H.C.F. of any two prime or coprime numbers equals 1.
10. The L.C.M. of any two prime or coprime numbers equals their product.
11. The H.C.F. of two or more numbers is never greater than any of the numbers.
12. The L.C.M. of two or more numbers is never less than any of the numbers.
13. The H.C.F. of two or more numbers is a factor of their L.C.M.
14. A number is divisible by
    - (i) 2, if the unit's digit of the number is 0, 2, 4, 6 or 8.
    - (ii) 3, if the sum of the digits is divisible by 3.
    - (iii) 4, if the number formed by its digits in ten's and unit's places is divisible by 4.

(iv) 5, if unit's digit is 0 or 5.

(v) 6, if it is divisible by both 2 and 3.

(vi) 8, if the number formed by its digits in hundred's, ten's and unit's places is divisible by 8.

(vii) 9, if the sum of the digits is divisible by 9.

(viii) 10, if the unit's digit is 0.

(ix) 11, if the difference of the sum of its digits in odd places and the sum of its digits in even places (starting from unit's place) is either 0 or divisible by 11.

CHAPTER FIVE

Integers

In this chapter, we shall extend our number system from whole numbers to integers, represent integers on the number line, discuss the idea of absolute value of an integer, operations on integers and their properties and study what are called powers of integers.

5.1 The Need for Integers and Their Introduction

In the earlier chapters, we have studied whole numbers and operations on them. We saw that it is not always possible to subtract a number from another in whole numbers. For example, we have no whole number to represent $6-10$, $10-17$, etc. Therefore, there is a need to extend our number system so that we have numbers to represent such differences.

Let us consider natural numbers, i.e. 1, 2, 3, 4, 5, . . . and for each natural number (whole number other than zero) we create a new number as follows:

> For 1 we create -1 (called *negative* one or *minus* one), such that $1+(-1)=0$. -1 and 1 are called *opposites* of each other.
> For 2 we create -2 (called negative two or minus two), such that $2+(-2)=0$. -2 and 2 are called opposites of each other.
> For 3 we create -3 (called negative three or minus three), such that $3+(-3)=0$. 3 and -3 are called opposites of each other, and so on.

Thus, our new collection together with whole numbers becomes 0, 1, $-1, 2, -2, 3, -3 \ldots$. We rewrite this as

$$\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$$

These numbers are called *integers.* The numbers 1, 2, 3 . . . , i.e. natural numbers are called *positive integers* and -1, -2, -3 . . . are called *negative integers.*

The number 0 is simply an integer — it is neither positive nor negative. Positive integers are also written as $+1$, $+2$, $+3$ . . ., however, the plus (+) sign is usually omitted and understood.

A point to note here is that whenever opposites are involved simultaneously in a real situation, it is necessary to use integers to express the situation in mathematical terms. For example, *profits* and *losses* of a shopkeeper during certain period, heights of places expressed in terms of distances *above* and *below* sea level, temperatures of objects or places in terms of degrees *above* and below 0°C, etc., all need positive and negative integers for their representation. In such situations, one can represent profits, heights above sea level, temperatures above 0°C, etc. by positive integers and their opposites, namely losses, heights below sea level, temperatures below 0°C, etc. by negative integers.

Remark: We use the symbol '$-$' to denote negative integers. We also use this symbol to indicate subtraction. The context will make it clear whether we mean negative integer or subtraction. For instance, when we say that the temperature at Srinagar was -8°C, it is immediately clear that no subtraction is involved and that the negative integer '-8' is indicated. On the other hand, when we say, 'determine 46 − 17' it is immediately clear that subtraction of 17 from 46 is required.

5.2 Representation of Integers on Number Line

Since negative integers are opposites of positive integers, viz. natural numbers, we represent them in the opposite direction, i.e. on the left side of zero on the number line. The opposite integers (e.g. 3 and -3) are kept at the same distance from zero which is regarded as neither positive nor negative. Thus, we have the integers on the number line as shown in Fig. 5.1.

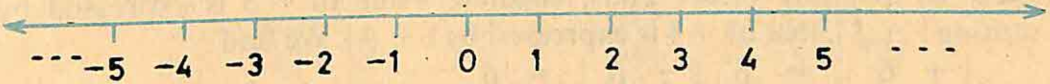


Fig. 5.1

5.3 Ordering of Integers

Let us carefully observe the number line given in Fig. 5.1 above. We recall that every whole number on the right of the number line was regarded as greater than every *whole number* to its left. Here also we follow the same *convention.* We find

$2 > 1$ since 2 is to the right of 1.
$1 > 0$ since 1 is to the right of 0.
$0 > -1$ since 0 is to the right of -1.
$-1 > -2$ since -1 is to the right of -2.

We conclude:

(i) *Every positive integer is greater than every negative integer.*
(ii) *Zero is less than every positive integer.*
(iii) *Zero is greater than every negative integer.*
(iv) *The greater the number, the lesser is its opposite.*
For example, $5 > 2$, $-5 < -2$, $-3 > -5$, $3 < 5$. In other words, *If* a *and* b *are two integers such that* $a > b$, *then* $-a < -b$.

5.4 Absolute Value of an Integer

The absolute value of an integer is the numerical value of the integer regardless of its sign. Thus, the absolute value of an integer is always positive. On the number line it is regarded as the distance of the integer from 0 irrespective of its direction. Thus, the absolute value of

$+5$ is 5,
-5 is 5,
$+7$ is 7,
-7 is 7,
0 is 0.

The absolute values of integers are essential in the further study of integers. We use two vertical lines, one on either side of the integer to show its absolute value. Thus, absolute value of -3 is expressed by writing $|-3|$, that of $+4$ is expressed by $|+4|$. We find

$$|+6| = 6 \quad |-6| = 6$$
$$|-11| = 11 \quad |+11| = 11$$

We conclude:

If a represents an integer,
$|a| = a$ *if a is positive or zero*
$|a| = -a$ *if a is negative.*

EXERCISE 5.1

1. Give the opposites of
    (i) Increase in population (ii) Depositing money in a bank
    (iii) Earning money (iv) Going east
    (v) 12 (vi) -2
2. Indicate the following by using integers:
    (i) 3° above zero (ii) 5° below zero
    (iii) A withdrawal of Rs 25 from an account (iv) A deposit of Rs 100 in an account
3. Which number in each of the following pairs is to the right of the other on the number line?
    (i) 1, 7 (ii) $-2, -5$ (iii) $0, -3$ (iv) $-5, 8$
4. Using the number line, write the integer which is
    (i) 4 more than 3 (ii) 5 less than 2
    (iii) 8 more than -9 (iv) 7 less than -3
5. Which number in each of the following pairs is smaller?
    (i) $8, -8$ (ii) $0, -12$
    (iii) $-15, -5$ (iv) $318, -356$
6. Write all integers between
    (i) -5 and 2 (ii) 0 and 4
    (iii) -4 and 4 (iv) -7 and 0

7. Replace * in each of the following by $<$ or $>$ so that the statement is true:
   - (i) $0 * 5$
   - (ii) $-7 * -17$
   - (iii) $-3 * 0$
   - (iv) $-81 * 18$
   - (v) $-13 * 13$
   - (vi) $-253 * -523$
8. Write the absolute value of each of the following:
   - (i) 17
   - (ii) -23
   - (iii) 0
   - (iv) -107
   - (v) -245
   - (vi) 1024
   - (vii) $(x-2)$ if x is greater than 2
   - (viii) $(x-2)$ if x is less than 2
9. Which of the following statements are true?
   - (i) The smallest integer is zero.
   - (ii) The opposite of zero is zero.
   - (iii) -18 is greater than -5.
   - (iv) A positive integer is greater than its opposite.
   - (v) Zero is not an integer since it is neither positive nor negative.
   - (vi) The absolute value of an integer is always greater than the integer.

5.5 Addition of Integers

We know how to add two whole numbers. Since each whole number is either a positive integer or zero, we can add the integers if they are positive or zero. But what happens if both of them or one of them is negative? We should like to have a rule or rules for addition of integers which would give us answers consistent with our everyday experiences. For example, we know that a profit of Rs 5 and a loss of Rs 2 result in a net profit of Rs 3. Therefore, we must have $5 + (-2) = 3$. A profit of Rs 3 and a loss of Rs 5 result in a net loss of Rs 2. Therefore, we must have $3 + (-5) = -2$. Similarly, a loss of Rs 3 and again a loss of Rs 2 result in a net loss of Rs 5. We, therefore, would like to have $(-3) + (-2) = -5$.

Let us go back to the number line and recall that $+3$ (or 3) represents a movement of 3 units to the right of zero and -3 represents a movement of 3 units to the left. With this background, let us represent some addition facts for integers on the number line.

Let us represent '2 + 4' on the number line. We begin at zero and move 2 units to the right. We reach at 2. We further move 4 units to the right and reach at 6 (see Fig. 5.2).

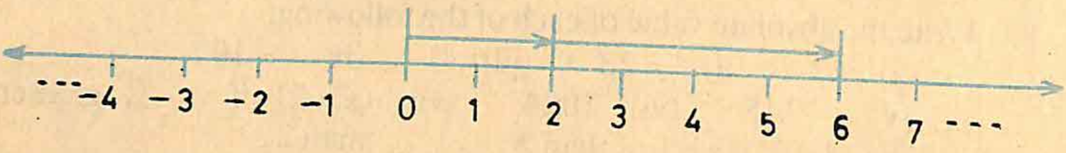


Fig. 5.2

Thus, $2 + 4 = 6$. We note that $|2| + |4| = 6$

Let us now represent '2 + 4' on the number line. We first move 2 units to the left of zero and then from there move 4 units to the right to account for + 4 (see Fig. 5.3). We reach at 2.

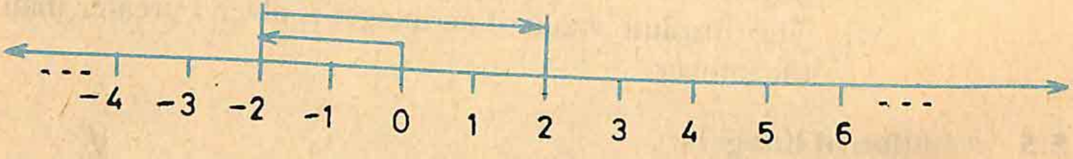


Fig. 5.3

Thus, $-2 + 4 = 2$. We note that $|4| - |-2| = 2$.

Finally, we represent $-2 + (-4)$ on the number line. We first move 2 units to the left of zero to account for -2 and further move 4 units to the left to account for -4. Thus, we reach at -6 (see Fig. 5.4). Therefore $(-2) + (-4) = -6$. We note that $|-2| + |-4| = 6$ and $-(|-2| + 1 -4|) = -6$.

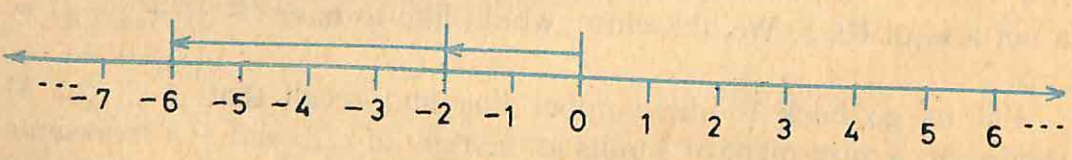


Fig. 5.4

In view of the above, we conclude:

(i) *To add two positive integers or two negative integers, we add their absolute values and assign the sign of the addends to the sum.*

Example 1: Add -2578 and -636.

Solution: We observe that both the integers are negative. Therefore, we use rule (i) mentioned above.

$|-2578| = 2578 \qquad |-636| = 636$

$-2578 + (-636) = -(2578 + 636) = -3214$

(ii) *To add a positive and a negative integer, we determine the difference of their absolute values and assign the sign of the addend having greater absolute value.*

Example 2: Add 384 and -905.

Solution: The integers are of unlike signs. Therefore, to find their sum we use rule (ii), mentioned above.

$|384| = 384 \quad |-905| = 905$

$$384 + (-905) = -(905 - 384)$$
$$= -521$$

5.6 Properties of Addition of Integers

When we extend our number system from whole numbers to integers, all properties of operations on whole numbers remain intact. There may be one or two additional properties. It is so because all whole numbers are also integers. We briefly state the properties of addition as follows where a, b, c represent integers.

Property I: *$a + b$ is an integer.*

Examples: (i) $2 + (-7) = -5$, -5 is an integer

(ii) $-3 + (-8) = -11$, -11 is an integer.

(iii) $17 + (-6) = 11$, 11 is an integer.

Property II: $a + b = b + a$

Examples: (i) $3 + (-4) = (-4) + 3$ since each equals (-1).

(ii) $(-3) + (-9) = (-9) + (-3)$ since each equals (-12).

Property III: $a + (b + c) = (a + b) + c$

Examples: (i) $[(-3) + (-2)] + 5 = (-3) + [(-2) + 5]$

Since $[(-3) + (-2)] + 5 = (-5) + 5 = 0$

and $(-3) + [(-2) + 5] = (-3) + 3 = 0$

(ii) $[(-1) + 4] + (-6) = (-1) + [4 + (-6)]$

Since $[(-1) + 4] + (-6) = 3 + (-6) = -3$

and $(-1) + [4 + (-6)] = (-1) + (-2) = -3$

As before, we usually write $a + b + c$ for the equal sums mentioned above.

In view of Properties II and III *even if we rearrange a given collection of three or more integers their sum is not changed.*

Property IV: $a + 0 = 0 + a = a$

Property V: We recall that if we add 1 to any whole number, we get the successor of that whole number. In case of integers also the same property holds. However, in case of whole numbers, 0 was not the successor of any whole number. In the case of integers, 0 is the successor of -1, -1 is the successor of -2, and so on.

Example 3: Find the sum of -21, -9, 63, -22 and -28.

Solution: $(-21) + (-9) + 63 + (-22) + (-28)$.

$= [(-21) + (-9)] + 63 + [(-22) + (-28)]$

$= -30 + 63 + (-50)$

$= [-30 + (-50)] + 63$

$= -80 + 63$

$= -17$

EXERCISE 5.2

1. Draw a number line and represent each of the following on it:

(i) $-6 + 8$ (ii) $5 + (-9)$

(iii) $-3 + (-8)$ (iv) $-1 + (-2) + 2$

(v) $-2 + 7 + (-8)$ (vi) $-2 + (-3) + (-5)$

2. Add the integers in each of the following:

(i) $-245, 111$	(ii) $2567, -3$
(iii) $10001, -2$	(iv) $-99005, 360$
(v) $-498, -320$	(vi) $-5894, 0$
(vii) $3003, -999$	(viii) $2884, -2884$
(ix) $2547, -2548$	(x) $-623, -5832, 623$
(xi) $-982, 1934, -18, -2034$	
(xii) $-4329, 4648, 4371$	

3. Find the sum:
   (i) $100 + (-66) + (-34)$
   (ii) $1262 + (-366) + (-962) + 566$
   (iii) $908 + (-8) + (-1) + 1 - 300$
   (iv) $-391 + (-81) + 9 + (-18)$
   (v) $373 + (-245) + (-373) + 145 + 3000$
   (vi) $1 + (-475) + (-475) + (-475) + (-475) + 1900$

4. Let us invent an operation $*$ for integers such that for two integers a and b

$$a * b = a + b + (-1)$$

For example, $2 * 3 = 2 + 3 + (-1) = 4$. Determine:
   (i) $3 * (-4)$ (ii) $15 * (-1)$

5. Which of the following statements are true?
   (i) The sum of a number and its opposite is zero.
   (ii) The sum of two negative numbers is a positive number.
   (iii) The sum of a negative number and a positive number is always a negative number.
   (iv) The successor of -297 is -298.
   (v) The sum of three different integers can never be zero.

5.7 Subtraction of Integers

The positive and negative integers suggest pairs of opposite numbers, for instance, 1 and -1, 2 and -2, 3 and -3, etc. We observe that the sum in each pair is zero, namely, $1 + (-1) = 0$, $2 + (-2) = 0$, etc. In any such pair each integer is called the *negative* (or *additive inverse*) of the other.

Thus, *for each non-zero integer, a, there is an integer '$-a$' such that $a + (-a) = 0$. '$-a$' is called the negative (or additive inverse) of a. Negative of zero is zero itself.*

We recall that subtraction is a process inverse to that of addition. For instance, to subtract 3 from 7 is the same as to find a number which added to 3 gives 7.

Suppose we want to subtract '-3' from 4. It means we want to find a number which added to '-3' gives 4. Obviously, such a number is 7. Let us see how we do it with the help of number line. To subtract '-3' from 4 suggests that if we are 3 steps to the left of starting point, i.e. 0, we need to determine the number of steps required to reach 4. From Fig. 5.5, we see that the number of steps required is 7. Therefore,

$$4 - (-3) = 7$$

Also, $4 + 3 = 7$

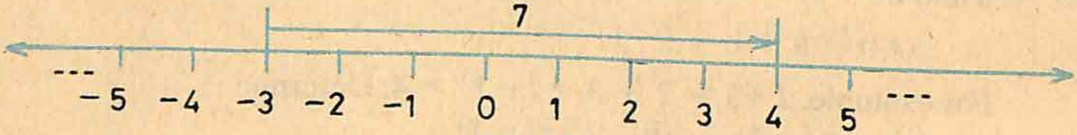


Fig. 5.5

Thus, we see that to subtract '-3' from 4, we add the negative (or additive inverse) of '-3' to 4. In fact this is the rule for subtraction of integers. The rule states:

If a and b are two integers, to subtract b from a, we change the sign of b and add it to a, i.e.

$$a - b = a + (-b)$$

Thus, we use $+(-b)$ and $-b$ interchangeably.

Example 4: Subtract '-13' from '-5'.

Solution: We change the sign of '-13' and add to '-5'.

Thus, $-5 - (-13) = -5 + 13 = 8$

In view of the above discussions and various properties of addition we are now in a position to find the value of an expression containing various terms with plus and minus signs as follows:

Step 1: *We add all terms with plus signs together.*

Step 2: *We add all terms with minus signs together.*

Step 3: *We find the difference of the absolute values of the two sums obtained in Steps 1 and 2.*

Step 4: *We assign to the result of Step 3 the sign of the sum having larger absolute value.*

Example 5: Find the value of

$$-12 + (-98) - (-84) + (-7)$$

Solution: We rewrite the given expression as

$$\begin{aligned} -12 - 98 + 84 - 7 &= (-12 - 98 - 7) + 84 \\ &= -117 + 84 \\ &= -33 \end{aligned}$$

5.8 Properties of Subtraction

Property I: We have seen that the difference of any two integers is also an integer. In other words, *if a and b are two integers and a − b = c, then c must be an integer.*

Note that this property does not hold for whole numbers.

Property II: In whole numbers 0 had no predecessor. In integers, −1 is the predecessor of 0. Thus, if a is any integer, 'a − 1' is its predecessor.

Property III: As in the case of whole numbers, if a is an integer, a − 0 = a.

EXERCISE 5.3

1. Subtract the first integer from the second in each of the following:
   (i) 3, 8 (ii) 10, −4 (iii) −15, 10 (iv) −200, −100

(v) $1001, 101$ (vi) $2, -7$ (vii) $-812, 3126$
(viii) $8650, -6$ (ix) $-3987, -4109$ (x) $-155, 0$
(xi) $0, -1005$ (xii) $83241, 40321$

2. Subtract -5 from 7. Subtract 7 from -5. Are the two results the same?
3. Replace ∗ by '<' or '>' in each of the following to make the statement true:
   (i) $(-6) + (-9) * (-6) - (-9)$
   (ii) $(-12) - (-12) * (-12) + (-12)$
   (iii) $(-20) - (+20) * 20 - (+65)$
4. Fill in the blanks:
   (i) $-6 +$ —— $= 0$
   (ii) $19 +$ —— $= 0$
   (iii) $12 + (-12) =$ ——
   (iv) $-4 +$ —— $= 12$
   (v) $-256 +$ —— $= -396$
   (vi) —— $- 215 = -64$
5. The sum of two integers is 48. If one of the integers is -24, determine the other.
6. The sum of two integers is -396. If one of them is 64, determine the other.
7. Find the value of
   (i) $-17 - (-13)$
   (ii) $-7 - 8 - (-25)$
   (iii) $(2 - 3) + (2 - 3)$
   (iv) $-13 + 32 - 18 - 1$
   (v) $50 - (-48) - (-2)$
   (vi) $-7 + (-8) + (-90)$
   (vii) $18 - [(-3) + 15]$
   (viii) $-12 - [(-15) + (-2) - 3]$
8. p and q are two integers such that p is the predecessor of q. Find the value of $P - q$.
9. If Δ is an operation such that for integers a and b, $a \Delta b = -a + b - (-2)$, find the value of

(i) $4 \Delta 3$ (ii) $(-2) \Delta (-3)$ (iii) $6 \Delta (-5)$ (iv) $(-5) \Delta 6$

10. Which of the following statements are true?
    - (i) $-13 > -8 - (-2)$
    - (ii) $-4 + (-2) < 2$
    - (iii) The negative of a negative integer is a positive integer.
    - (iv) If a and b are two integers such that $a > b$ then $a - b$ is always a positive integer.
11. Calculate
    $1 - 2 + 3 - 4 + 5 - 6 + \ldots + 19 - 20$
12. Calculate the sum
    $2 + (-2) + 2 + (-2) + 2 + (-2) + \ldots$
    - (i) if the number of twos (terms) is 319.
    - (ii) if the number of twos is 230.

5.9 Multiplication of Integers

We recall that multiplication is repeated addition. For instance,
$3 \times 5 = 5 + 5 + 5$ or 15.
In the same way we can find the product of any two integers. Thus,

$$\begin{aligned} 4 \times (-5) &= (-5) + (-5) + (-5) + (-5) \\ &= -20 \\ &= -(4 \times 5) \end{aligned}$$

Let us now consider the product $(-4) \times 3$. We have

$$\begin{aligned} (-4) \times 3 &= 3 \times (-4) \qquad \text{(Why?)} \\ &= -4 + (-4) + (-4) \\ &= -12 \\ &= -(3 \times 4) \end{aligned}$$

In view of the above, we conclude:

To find the product of a positive and a negative integer, we find the product of their absolute values and assign minus sign to the product.

Let us now find the product of two integers with like signs, i.e. either both the integers are positive or both of them are negative.

If both the integers are positive, they are whole numbers and the product is obtained as in the case of whole numbers.

Let us consider the case when both the integers are negative. Let the integers be '-3' and '-2'. To do this, let us observe the following table:

$$\begin{aligned}(-3)\times 4 &= -12\\ (-3)\times 3 &= -9\\ (-3)\times 2 &= -6\\ (-3)\times 1 &= -3\\ (-3)\times 0 &= 0\\ (-3)\times(-1) &= ?\end{aligned}$$

We observe that as the second integer decreases by 1, the product increases by 3. Therefore, the product $(-3)\times(-1)$ should be 3 more than 0, i.e. 3. Similarly, $(-3)\times(-2)$ should be 3 more than 3, i.e. 6, and so on. We have, thus, found

$$\begin{aligned}(-3)\times(-2) &= 6\\ &= +(3\times 2)\end{aligned}$$

The distributive property of multiplication over addition (which also holds good in integers) also helps us in getting the same result. We have

$$\begin{aligned}&(-3)\times[2(-2)] = (-3)\times(0)\\ \text{or, }&(-3)\times 2+(-3)\times(-2)=0\\ \text{or, }&-6+(-3)\times(-2)=0\end{aligned}$$

This shows that $(-3)\times(-2)$ is the integer which when added to '-6' gives the sum zero. We know that 6 added to '-6' gives zero as the sum. Therefore,

$$(-3)\times(-2) = 6 = +(3\times 2)$$

In view of the above, we conclude:

The product of two integers, both positive or both negative, is a positive integer equal to the product of the absolute values of the integers.

Example 6: Determine each of the following products:

(i) $(-15)\times(-18)$ (ii) $(-23)\times 7$

Solution: (i) $(-15)\times(-18) = +(15\times 18)$

$$= 270$$

(ii) $(-23)\times 7 = -(23\times 7)$

$$= -161$$

5.10 Properties of Multiplication

All properties of multiplication of whole numbers also hold *for* integers. We restate the properties in brief as follows wherein a, b, c are integers.

Property I: $a \times b$ *is an integer*

Examples: (i) $2 \times (-3) = -6$, -6 is an integer.

(ii) $(-3) \times (-7) = 21$, 21 is an integer.

Property II: $a \times b = b \times a$

Examples: (i) $3 \times (-4) = (-4) \times 3$ as each equals (-12).

(ii) $(-6) \times (-7) = (-7) \times (-6)$ as each equals 42.

Property III: $a \times (b \times c) = (a \times b) \times c$

Examples: (i) $(-2) \times [3 \times (-5)] = [(-2) \times 3] \times (-5)$

Since $(-2) \times [3 \times (-5)] = (-2) \times (-15) = 30$

and $[(-2) \times 3] \times (-5) = (-6) \times (-5) = 30$

(ii) $(-4) \times [(-5) \times (-6)] = [(-4) \times (-5)] \times (-6)$

Since $(-4) \times [(-5) \times (-6)] = (-4) \times (30) = -120$

and $[(-4) \times (-5)] \times (-6) = (20) \times (-6) = -120$

As before, we write $a \times b \times c$ for the equal products mentioned above.

In view of Properties II and III *even if we rearrange the integers in a product of 3 or more integers, it does not change the product.*

We also observe that

(i) When the number of negative integers in a product is odd, the product is negative.

(ii) When the number of negative integers in a product is even, the product is positive.

For example, each of the products $(-3) \times (-2) \times (-3) \times 4$, $(-1) \times 7 \times 8$ and $(-4) \times (-5) \times (-6) \times (-7) \times (-3)$ must be negative and each of the products $(-2) \times (-5)$, $(-2) \times (-6) \times 3$ and $(-7) \times (-8) \times (-5) \times (-9) \times 7 \times 4$ must be positive.

Property IV: $a \times 0 = 0 \times a = 0$

Property V: $a \times 1 = 1 \times a = a$.

We have the following additional property.

Property VI: $a \times (-1) = (-1) \times a = -a$

We know that a and $-a$ are opposties of each other. Thus, to find the negative of an integer, we need to multiply the integers by (-1).

Property VII: (i) $a \times (b + c) = a \times b + a \times c$

(ii) $a \times (b - c) = a \times b - a \times c$

Examples: (i) $(-3) \times [(-5) + 4] = (-3) \times (-5) + (-3) \times 4$

Since $(-3) \times [(-5) + 4] = (-3) \times (-1) = 3$

and $(-3) \times (-5) + (-3) \times 4 = 15 - 12 = 3$

(ii) $(-6) \times [(-2) - 5] = (-6) \times (-2) - (-6) \times 5$

Since $(-6) \times [(-2) - 5] = (-6) \times (-7) = 42$

and $(-6) \times (-2) - (-6) \times 5 = 12 - (-30) = 42$

EXERCISE 5.4

1. Find each of the following products:

(i) $2 \times (-15)$

(ii) $(-225) \times 8$

(iii) $(-17) \times (-20)$

(iv) $3 \times (-8) \times 5$

(v) $9 \times (-3) \times (-6)$

(vi) $(-12) \times (-12) \times (-12)$

(vii) $(-2) \times 36 \times (-5)$

(viii) $(-8) \times (-43) \times 0$

(ix) $18 \times (-185) \times (-4)$

(x) $(-45) \times 55 \times (-10)$

(xi) $(-1) \times (-2) \times (-3) \times (-4) \times (-5)$

(xii) $(-3) \times (-6) \times (-9) \times (-12)$

2. Complete the following Multiplication Table:

Second Number

First Number

X	− 4	− 3	− 2	− 1	0	1	2	3	4
− 4									
− 3									
− 2									
− 1									
0									
1									
2									
3									
4									

Is the Multiplication Table symmetrical about the diagonal joining the upper left corner to the lower right corner?

3. Find the value of
   - (i) $(-8) \times 0 \times 37 \times (-37)$
   - (ii) $1569 \times 887 - 569 \times 887$
   - (iii) $(-183) \times (-44) + (-183) \times (-56)$
   - (iv) $18946 \times 99 - (-18946)$
   - (v) $15625 \times (-2) + (-15625) \times 98$
   - (vi) $(-8) \times (10 - 5 - 43 + 98)$
4. What will be the sign of the product if we multiply together
   - (i) 8 negative integers and 1 positive integer?
   - (ii) 21 negative integers and 3 positive integers?
5. Determine the integer whose product with '− 1' is
   (i) − 40 (ii) 46 (iii) 0

6. Compare, i.e. state which is greater:
    (i) $(8 + 9) \times 10$ and $8 + 9 \times 10$
    (ii) $(8 - 9) \times 10$ and $8 - 9 \times 10$
    (iii) $[(-2) - (5)] \times (-6)$ and $(-2) - 5 \times (-6)$
7. Which of the following statements are true?
    (i) The product of three negative integers is a negative integer.
    (ii) Of the two integers, if one is negative, then their product must be negative.
    (iii) The product of a negative and a positive integer may be zero.
    (iv) There does not exist an integer b such that for $a > 1$

    $$a \times b = b \times a = b$$

    (v) For all non-zero integers a and b, $a \times b$ is always greater than either a or b.

5.11 Division of Integers

We know that division is inverse of multiplication. Each multiplication fact gives rise to corresponding division fact(s). For example, the multiplication fact '$2 \times 3 = 6$' gives the following division facts:

$$6 \div 2 = 3 \text{ and } 6 \div 3 = 2$$

In the case of integers we have $(-4) \times 5 = -20$ and the corresponding division facts are:

$$-20 \div (-4) = 5 \text{ and } -20 \div 5 = -4$$

As usual, the number to be divided is called *dividend* and the number which divides is called the *divisor.* The result of division is called *quotient.* In the above example, we note that *when dividend is negative and divisor is negative, the quotient is positive. When the dividend is negative and divisor is positive, the quotient is negative.* Let us consider one more example and see whether or not the same thing happens. We take the multiplication fact $(-5) \times (-8) = 40$. We get

Corresponding Division Fact	*Dividend*	*Divisor*	*Quotient*
$40 \div (-5) = -8$	Positive	Negative	Negative
$40 \div (-8) = -5$	Positive	Negative	Negative

In view of the above we conclude:

(i) *The quotient of two integers both positive or both negative is a positive integer equal to the quotient of the corresponding absolute values of the integers.*

(ii) *The quotient of a positive and a negative integer is a negative integer and its absolute value is equal to the quotient of the corresponding absolute values of the integers.*

As usual, the divisor is a non-zero integer. In other words, *division by zero is not defined.*

Example 7: Divide
(i) 68 by -17 (ii) -78 by 13 (iii) (-60) by (-12)

Solution: (i) $|68| = 68, |-17| = 17$
We have $68 \div 17 = 4$
Therefore, $68 \div (-17) = -4$
(ii) $(-78) \div 13 = -(78 \div 13) = -6$
(iii) $(-60) \div (-12) = +(60 \div 12) = 5$

5.12 Properties of Division

Property I: *If a and b are two integers,* $a \div b$ is not always an integer. For example, none of $(-4) \div 5$, $5 \div (-3)$, $1 \div 2$ is an integer.

Property II: For any integer a,
$a \div a = 1\ (a \neq 0)$ $a \div 1 = a$

Property III: For any integer a,

$a \div (-1) = -a, a \div (-a) = -1, (a \neq 0)$

$(-a) \div a = -1 \ (a \neq 0)$

Property IV: For any non-zero integer a, $0 \div a = 0$.

EXERCISE 5.5

1. Find the quotient in each of the following:

(i) $18 \div (-3)$

(ii) $(-18) \div 3$

(iii) $(-18) \div (-3)$

(iv) $36 \div (-9)$

(v) $(-48) \div (-16)$

(vi) $0 \div (-12)$

(vii) $(-1728) \div 12$

(viii) $-15625 \div (-125)$

(ix) $(-729) \div (-81)$

(x) $10569 \div (-1)$

(xi) $200000 \div (-100)$

(xii) $17699 \div (-17699)$

2. Fill in the blanks:

(i) $296 \div$ —— $= 296$

(ii) $-3785 \div$ —— $= 1$

(iii) —— $\div 578 = 0$

(iv) —— $\div 1 = -3065$

(v) —— $\div 156 = -2$

(vi) —— $\div 567 = -1$

5.13 Use of Brackets

In order to simplify a numerical expression with two or more of the fundamental operations, we follow the conventions given below:

(i) *Generally we perform operations sequentially from left to right in the order division, multiplication, addition, subtraction.*

For example,

(a) $36 \div 6 + 3 = 6 + 3 = 9$

(b) $12 - 4 \div 2 = 12 - 2 = 10$

(c) $55 - 7 \times 3 + 35 - 4$

$= 55 - 21 + 35 - 4$

$= 55 + 35 - 21 - 4$

$= 90 - 21 - 4$

$= 69 - 4$

$= 65$

(ii) When the operations of division and multiplication occur simultaneously (as in $36 \div 6 \times 3$) without an operation of addition or subtraction between them, which of the two operations, multiplication or division, is to be performed first, will be indicated by ( ). Thus, $36 \div (6 \times 3)$ shows that first the operation of multiplication is to be performed.

(iii) Many a time we come across situations where we need to be certain to indicate definitely which operations are to be performed first. To do this we use brackets (grouping symbols). For example, if we want to divide 36 by the sum of 2 and 4, we indicate the sum of 2 and 4 by $(2 + 4)$ and write it as

$$36 \div (2 + 4) = 36 \div 6 = 6$$

If we do not use the symbol ( ), the expression becomes $36 \div 2 + 4$ which means $18 + 4$, i.e. 22. In many situations arrangement of terms within a second group has to be indicated. In such a case, we use different symbols so that we can easily know which terms are combined together. For example, when we say, '114 divided by the number 1 less than the product of 4 and 5', we treat '4×5' as one entity. We write (4×5) for this entity. We treat '$(4 \times 5) - 1$' as the second entity and write $\{(4 \times 5) - 1\}$ for this using another symbol '{ }' to indicate the new arrangement. Thus the expression becomes

$$114 \div \{(4 \times 5) - 1\}$$

Most commonly used grouping symbols are:

Symbols	*Names*
( )	Parentheses or common brackets
{ }	Braces or curly brackets
[ ]	Brackets or square brackets

The left part of each symbol indicates its start and the right part indicates its end.

(iv) In some cases we may use another symbol '—' called *bar / vinculum* which in order comes first. It occurs on the innermost terms and is placed like

$$36 \div (8 - \overline{4 + 2}).$$

In an expression, if more than one symbols are used, we first remove the innermost symbol by doing the operations involved and then proceed with the next innermost, and so on. While removing such symbols we keep in mind that we mean 'multiplication' when there is no sign of operation between a number and a grouping symbol. Thus,

$$4(6-2) = 4 \times (6-2)$$

If an arrangement within brackets is preceded by a '+' sign, to remove brackets, we do the operations as follows:

$$\begin{aligned} 9 + (7-3+2) &= 9 + 7 - 3 + 2 \\ &= 18 - 3 \\ &= 15 \end{aligned}$$

If an arrangement within brackets is preceded by a'−' sign and we want to remove brackets, then we change all the positive signs within brackets to negatives and vice versa. For example,

$$\begin{aligned} 9 - (7 - 3 + 2) &= 9 - 7 + 3 - 2 \\ &= 12 - 7 - 2 \\ &= 3 \end{aligned}$$

Let us now consider some examples that will make these ideas more clear.

Example 8: Find the value of

$$17 - [3 + \{18 - (19 - 2)\}]$$

Solution: $17 - [3 + \{18 - (19 - 2)\}]$

$= 17 - [3 + \{18 - 17\}]$

$= 17 - [3 + 1]$

$= 17 - 4$

$= 13$

Example 9: Simplify:

$(-13) + (-6) \div 2 - [(-5) \times (-4) - \{2 - (3 - 5)\}]$

Solution: The given expression is equal to

$(-13) + (-3) - [20 - \{2 - (-2)\}]$

$= (-13) + (-3) - [20 - 4]$

$= (-16) - (16)$

$= -32$

EXERCISE 5.6

1. Find the value of
   - (i) $24 + 15 \div 3$
   - (ii) $120 - 20 \div 2$
   - (iii) $12 - (3 \times 5) + 3$
   - (iv) $3 - (5 - 6 \div 3)$
   - (v) $28 - 5 \times 6 + 2$
   - (vi) $36 \div (5 + 7)$
   - (vii) $(-15) + 4 \div (5 - 3)$
   - (viii) $(-2) + (-8) \div (-4)$
   - (ix) $17 + (-3) \times (-5) - 6$
   - (x) $2 \div \overline{3 - 2}$
   - (xi) $(-40) \times (-1) + (-28) \div 7$
   - (xii) $(-5) - (-48) \div (-16) + (-2) \times 6$
2. Using brackets, write a mathematical expression for each of the following:
   - (i) Five multiplied by the sum of two and three
   - (ii) Twelve divided by the sum of one and three
   - (iii) Twenty divided by the difference of seven and two
   - (iv) Eight subtracted from the product of two and three
   - (v) Forty divided by one more than the sum of nine and ten
   - (vi) Two multiplied by one less than the difference of nineteen and six

3. Simplify:

(i) $20 + \{10 - 5 + (7 - 3)\}$
(ii) $81 \times [59 - \{7 \times 8 + (13 - 2 \times 5)\}]$
(iii) $121 \div [17 - \{15 - 3(7 - 4)\}]$
(iv) $3[18 + \{3 + 4(4 - 2)\}]$
(v) $(14 - 7) \times [8 + \{3 + \overline{7 - 1}\}]$
(vi) $18 + \{1 + (15 - 2) \times 4\}$
(vii) $2 - [2 - \{2 - (2 - \overline{2 - 2})\}]$
(viii) $118 - [121 \div (11 \times 11) - (-4) - \{3 - \overline{9 - 2}\}]$
(ix) $(-1)\{(-5) + (-25)\} \times (-7) - (8 - 10)(-4)$
(x) $15 - (-3)\{4 - \overline{7 - 3}\} \div 3[5 + (-3) \times (-6)]$

5.14 Powers of Integers

In mathematics we have to deal with a number of problems involving multiplication of a number by itself several times. We use a convenient notation to indicate this.

For	*We Write*	*We Read*
2×2	2^2	Two raised to two or two squared or second *power* of two
$2 \times 2 \times 2$	2^3	Two raised to three or two cubed or third power of two
$2 \times 2 \times 2 \times 2$	2^4	Two raised to four or fourth power of two
$2 \times 2 \times 2 \times 2 \times 2$	2^5	Two raised to five or fifth power of two

Thus, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, and so on. Obviously, $2^1 = 2$. In '2^5' the number '2' is called the *base* and '5' the *exponent*. 2^5, i.e. 32 is the *fifth power* of 2.

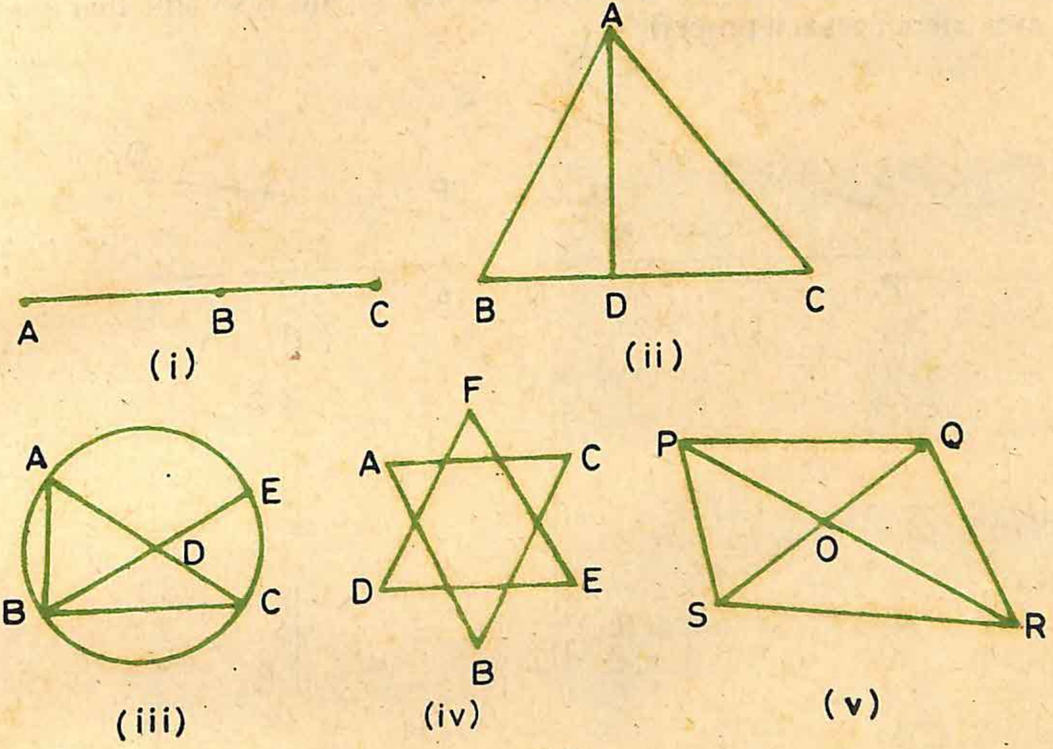
Similarly, $(-3)^2 = (-3) \times (-3) = 9$

$(-3)^3 = (-3) \times (-3) \times (-3) = -27$

$(-3)^4 = (-3) \times (-3) \times (-3) \times (-3) = 81$

EXERCISE 11.1

1. Count the number of line segments drawn in each of the following figures and name them:



(i) (ii) (iii) (iv) (v)

Fig. 11.2

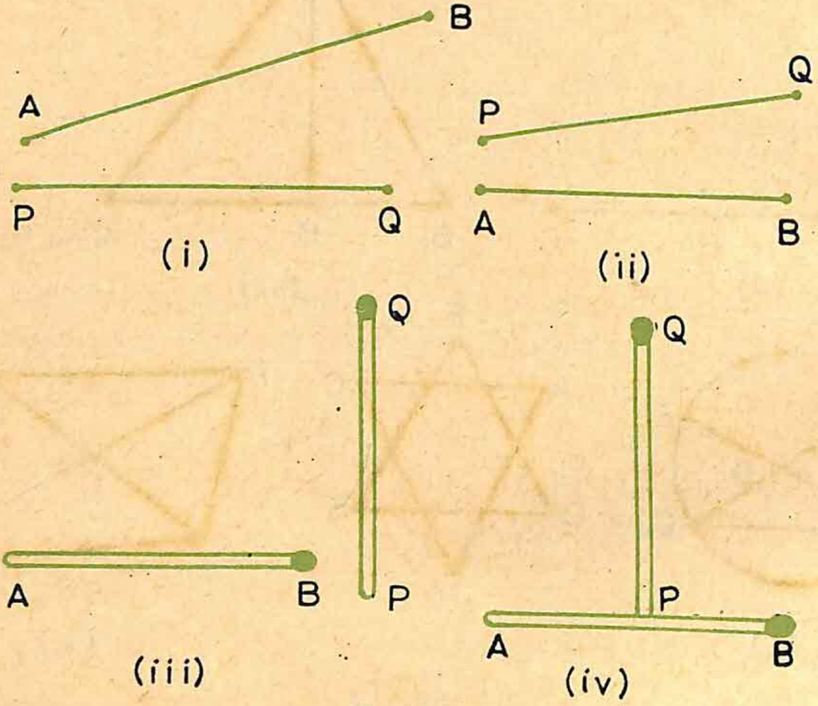
2. Mark two points on a paper and draw a line segment joining them. Name it.
3. What is the difference between a line and a line segment?
4. Give three examples of line segments from your environment.

11.2 Comparison of Line Segments

By comparison of two line segments we mean an (order) relation between their lengths, that is, which of them is longer or shorter.

(i) Comparison by Observation

Look at the two line segments AB and PQ in Fig. 11.3(i). Which of them is shorter? You can tell at once that PQ is shorter than AB. Now look at Fig. 11.3(ii). Can you tell at once which segment is longer? Probably not, because the difference in their lengths is so little that our eyes cannot get at it properly.



(i) (ii) (iii) (iv)

Fig. 11.3

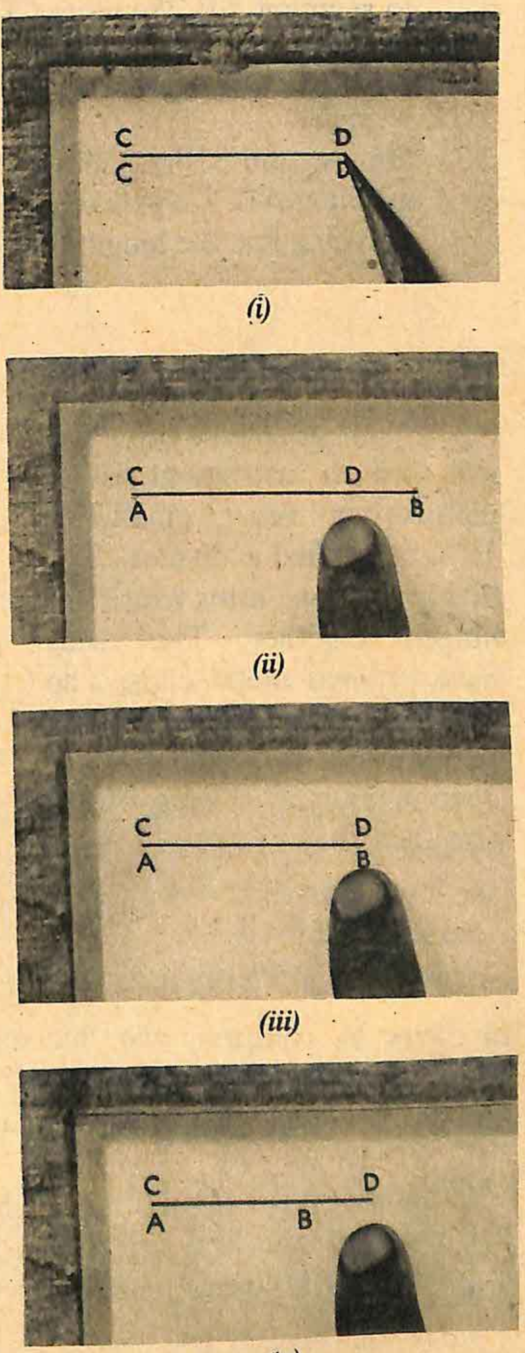
Let us look at the pair of match sticks (representing two line segments) in Fig. 11.3(iii) and Fig. 11.3(iv), respectively. Which of the two segments AB and PQ is longer? You will probably say PQ since it *appears* to be longer to our eyes. But this is only an optical illusion — segments standing upright appear to be longer than the horizontal segments. Actually segment AB equals segment PQ, but our sight has misled us. Therefore, we need better methods of comparing two segments.

(ii) Comparison by Tracing

Suppose we want to compare segments AB and CD. We take a tracing paper and place it on the segment CD [Fig. 11.4(i)]. We trace the segment on the tracing paper with a ruler and a pencil.

Now, we take this copy of the segment CD and place it on the segment AB so that C is placed on A and segment CD along segment AB. There are three possibilities for location of point D on line AB:

1. D is between A and B [Fig. 11.4(ii)]. We say that segment CD is shorter than segment AB. We write CD < AB and read 'CD is less than AB' meaning that the length of CD is less than the length of AB. Note that for the length of segment AB we use the symbol AB and for the length of segment CD we use the symbol CD, etc.
2. D is just on B [Fig. 11.4(iii)]. We say that segment CD is *equal*



(i)

(ii)

(iii)

(iv)

Fig. 11.4: Comparison by tracing

to segment AB. We write CD = AB and read 'CD is equal to AB' meaning that the length of CD is equal to the length of AB. Thus, *two line segments having the same length are said to be equal.*

3. D is beyond B [Fig. 11.4(iv)]. We say that segment CD is greater than segment AB. We write CD > AB and read 'CD is greater than AB', meaning that the length of CD is greater than the length of AB.

(iii) *Comparison by a Divider*

Open your geometry box. You will find an instrument whose picture has been given in Fig. 11.5. It is called a 'divider'. The divider has two arms which are hinged together. The arms have pointed metal ends. The distance between the ends can be changed or adjusted by opening out or closing in the arms.

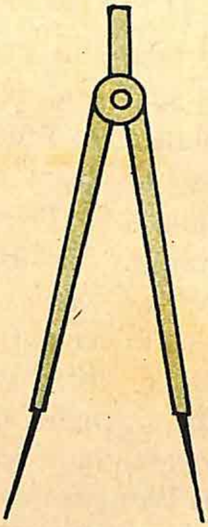
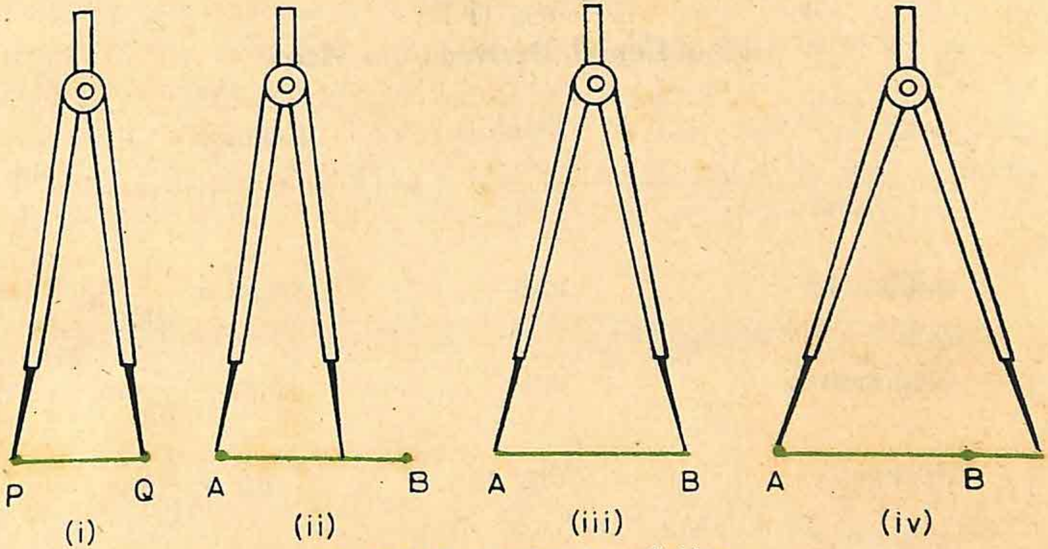


Fig. 11.5

In order to compare two line segments AB and PQ, we place the end-point of one arm of the divider on P and open the arms carefully so that the end-point of the other arm is at Q [Fig. 11.6(i)].

Now we lift the divider and without disturbing its opening, place the end-point of one arm on A and the end-point of the other arm on segment AB. If the end-point of the second arm falls

(i) between A and B, then PQ < AB [Fig. 11.6(ii)].



(i) (ii) (iii) (iv)

Fig. 11.6: Comparison by a divider

(ii) on B, then PQ = AB [Fig. 11.6(iii)].

(iii) beyond B, then PQ > AB [Fig. 11.6(iv)].

11.3 Measurement of Line Segments

We have seen three ways of comparing line segments. But for these, we have to observe or to place one line segment over the other. A better way of comparing line segments is to measure these line segments. A line segment is measured by comparing it with a standard segment which we refer as a *unit segment.* The number of times a unit segment is contained in the given segment is called its measure or *length.*

The basic unit of length in the international system of units (SI) is metre, written in short form as m. This unit was originally adopted by the French in 1791 just after the French Revolution and was supposed to be approximately equal to one ten-millionth part of the distance from the Equator to the North Pole. Metre is now defined as 1650763.73 wavelengths of the orange line in the spectrum of the Krypton isotopes 86 measured in vacuum and is equal to the distance between two fine marks on a platinum-irridium bar kept in a vault near Paris. Some of the multiples and sub-multiples of metre are given in Table 11.1.

TABLE 11.1
Unit of Length Derived from Metre

Unit	*Symbol*	*Relation*
millimetre	mm	$1\text{ mm} = \frac{1}{1000}\text{ m}$
centimetre	cm	$1\text{cm} = \frac{1}{100}\text{ m}$
decimetre	dm	$1\text{ dm} = \frac{1}{10}\text{ m}$
decametre	dam	1 dam = 10 m
hectometre	hm	1 hm = 100 m
kilometre	km	1 km = 1000 m

Note that 1 m = 100 cm, 1 cm = 10 mm, 1 m = 1000 mm.

Example: Convert 1.56 metres into centimetres.

Solution: $1.56\text{ m} = 1.56 \times 100\text{ cm}$ (Since 1 m = 100 cm)

$= 156\text{ cm}$

India has adopted the SI system of units since 1962. Prior to that we had the FPS (foot-pound-second) system of units. The basic unit of length in this system is a foot. The other common units of length in this system and their relation with SI units are as follows:

inch 1 in = 1″ = 0.0254 m = 2.54 cm
foot 1 ft = 1′ = 12″ = 0.3048 m = 30.48 cm
yard 1 yd = 3′ = 36″ = 0.9144 m
mile 1 mile = 1760 yd = 5280 ft = 1.608 km
1 m = 1.094 yd = 3.281 ft = 39.37 in
1 km = 0.6215 mile

For our class work, we use a straight edged ruler (an instrument of your geometry box) with centimetre and millimetre marks on one edge.

The opposite edge has inch marks and tenths of inch marks. Usually the length of this ruler is a foot or 6 inches. The marks on the ruler are called *graduations* and the ruler is called a *graduated* ruler.

In order to measure the length of a given line segment AB, we place the ruler with its edge along the segment AB such that the zero (0) mark of the ruler coincides with the point A (Fig. 11.7). Then we read the mark on the ruler which is against the point B. This mark indicates 5 big divisions (cm) and 8 small divisions (mm). So the length of line segment AB is 5 cm 8 mm or 5.8 cm.

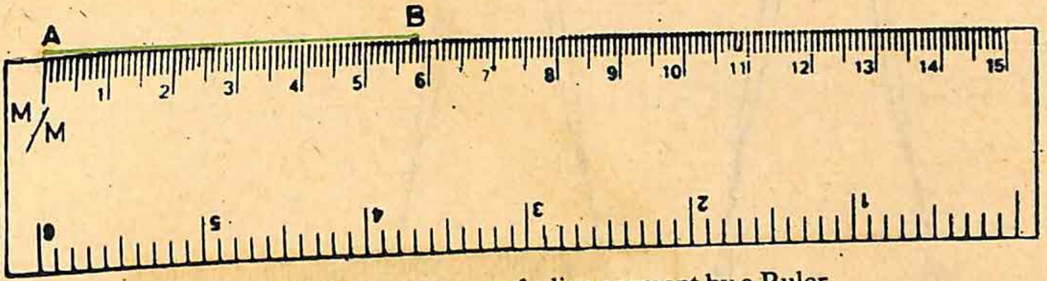


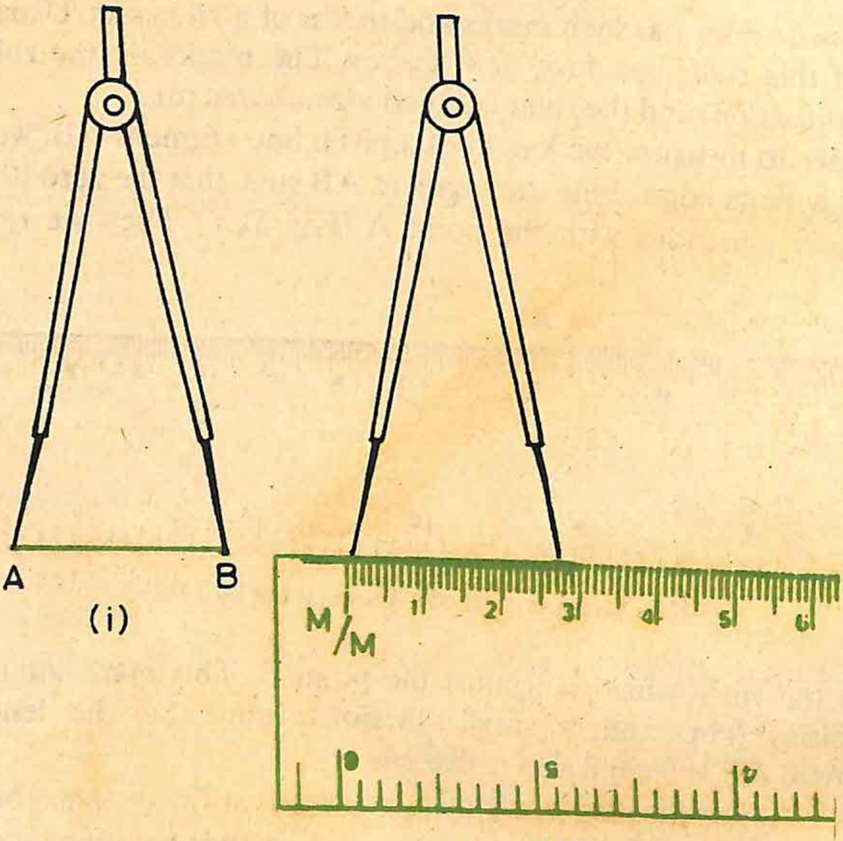
Fig. 11.7: Measurement of a line segment by a Ruler

Remarks: (i) For the above, we may also say that the distance between the points A and B is 5.8 cm. Thus, *the distance between two points equals the length of the line segment joining them.*

(ii) If it is not possible to use the zero mark, we can start our measurement from any other centimetre (cm) mark. Suppose we place the ruler so that 1 cm mark is against A then, in the above case, B would have to be taken against the mark for the 8 small divisions after the 6 cm mark.

The marks on the ruler are not at the same level as the segment AB because of the thickness of the ruler. This may cause some error in placing zero mark against A or reading the mark against B. To avoid this difficulty, we can measure line segments with the help of a divider as follows:

We open the divider so much that the end-point of one of its arms is at A and the end-point of the second arm is at B [Fig. 11.8 (i)]. Then we lift the divider and without disturbing its opening, place it on the



(i)

(ii)

Fig. 11.8: Measurement of a line segment by a Divider

ruler so that the end-point of one arm is at zero mark as shown in Fig. 11.8 (ii). We then read the mark against the end-point of the second arm of the divider. In the figure, it is 7 small divisions to the right of the big division mark 2. Thus, the length of the given segment AB is 2 cm 7 mm or 2.7 cm.

11.4 To Construct a Line Segment of Given Length

Suppose we have to construct a segment of length 3.8 cm. We mark a point A and then place the ruler so that zero mark of the ruler is at A. Mark with a pencil a point B against the mark on the ruler which indicates 3.8 cm (3 big divisions and 8 small divisions). Join points A and B by moving the tip of the pencil against the straight edge of the ruler (Fig. 11.9). Thus, we obtain the required segment AB of given length.

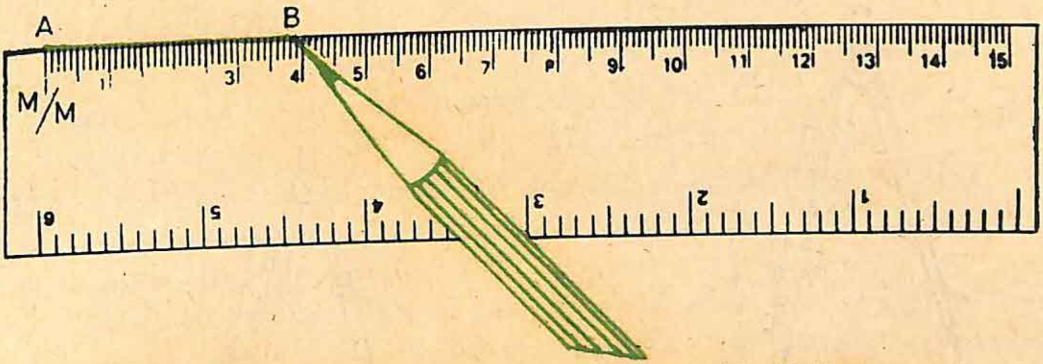


Fig. 11.9: Construction of a line segment with a Ruler

Line segments are also constructed by using an instrument called compasses. You can see this instrument in your geometry box. The compasses have two arms which are hinged together. One of the arms has a metal end-point. The other arm has a screw arrangement which can tightly hold a pencil whose end-point can be at an adjustable distance from the metal point (Fig. 11.10).

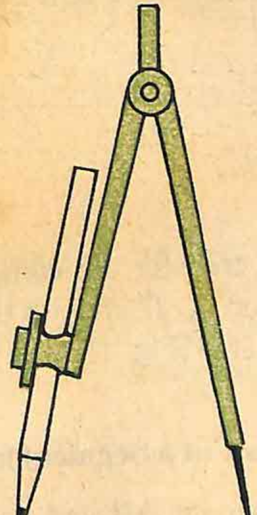
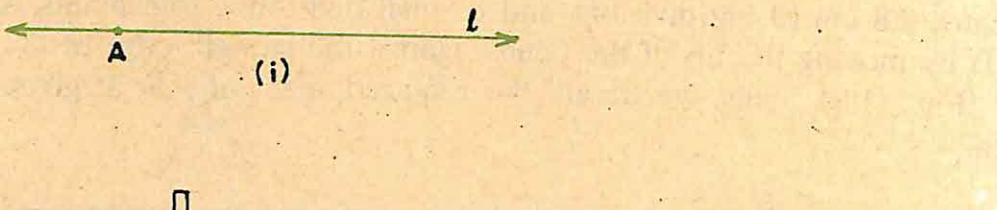


Fig. 11.10

To draw a line segment of length 3.8 cm, we mark a point A and draw a line, say *l*, passing through it [Fig. 11.11(i)]. Then we place the metal point of the compasses at zero mark on ruler and open out the compasses such that the pencil point is on the mark indicating 3.8 cm on the ruler [Fig. 11.11(ii)].



(i)

(ii)

(iii)

Fig. 11.11

Now, transfer the compasses as it is to the line *l*, so that the metal point is on A. Then with the pencil point make a small stroke on *l* so as to cut it at B [Fig. 11.11(iii)]. The segment AB has the required length of 3.8 cm.

11.5 To Cut a Segment from a Given Line Equal to a Given Segment

Let a segment AB and a point O on a line *l* be given (Fig. 11.12). We have to find a point X on *l* so that the segments AB and OX are equal. We can use a divider or compasses for this construction. Here, we are

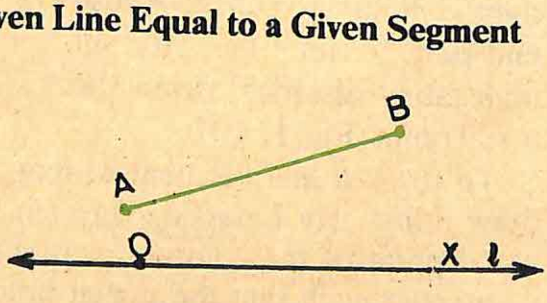


Fig. 11.12

using the compasses. Open out the compasses and adjust so that the metal point is on A and the pencil point is on B. Then transfer to the line *l*, the compasses without disturbing their opening so that the metal point is on O. Then with the pencil point make a small stroke on the line *l* to cut it at the point X. We thus obtain the required segment OX equal to a given segment AB.

Remark: If the line segment to be measured or constructed is very long, then longer measuring rulers are needed. Tailors use graduated metre tapes or rods (100 cm) to measure cloth.

11.6 To Construct a Segment XY whose Length is the Sum of the Lengths of Two Given Segments AB and CD

The construction is as follows:

We draw any line *l* and take any point X on it. Now, following the method of Section 11.5, we find a point P on *l* using compasses such that segment XP is equal to segment AB. Then we find a point Y on the line *l* such that segment PY is equal to segment CD as shown in Fig. 11.13.

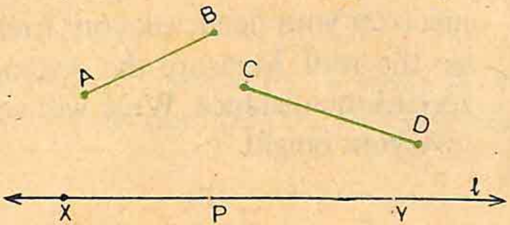


Fig. 11.13

Then XY is the required segment because the length of segment XY = XP + PY. In this case, we say that XY = AB + CD.

11.7 To Construct a Line Segment XY Whose Length is the Difference of the Lengths of Two Given Segments AB and CD (AB > CD)

The construction is as follows:

Let us take any point X on any line *l*. Then using compasses we find the point P on line *l* so that segment XP is equal to segment

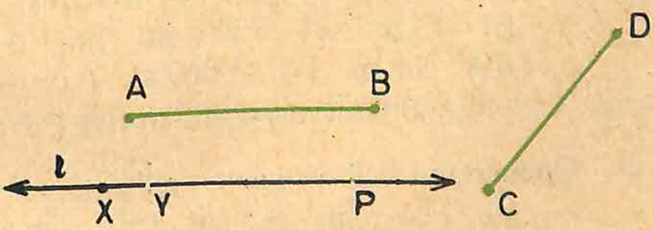


Fig. 11.14

AB. Next, we find the point Y so that segment PY is equal to segment CD as shown in Fig. 11.14. Then, XY is the required segment because XY = XP − PY. In this case, we say that XY = AB − CD.

Remarks: (i) There are two positions for Y on the line *l*. Here, we should take that position of Y which is on the same side of P as X. In the previous section, we took the other position of Y. (Why?)

(ii) How can we measure the lengths of very long line segments using a short ruler, say of length 15 cm? We may insert a number of points on the given segment breaking it up into shorter segments (each of which is less than or equal to 15 cm), measure each segment and add the lengths to get the required measurement.

Suggested Activity: Stand against a wall. With a pencil kept horizontal just over your head, ask your friend to mark the position of the pencil on the wall. Measure the distance of the mark from the floor and record the distance. What will you know from it? This distance will give your height.

EXERCISE 11.2

1. Compare the two segments given in Fig. 11.15 using a divider.

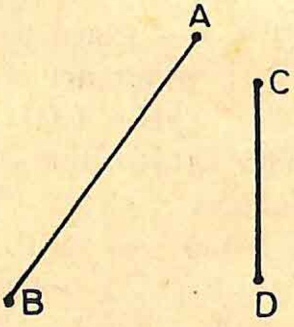


Fig. 11.15

2. Convert the following into centimetres:
   (i) 14 m (ii) 7 m 6 cm (iii) 5.06 m
   (iv) 5.8 m (v) 1.067 m (vi) 0.7 m
   (vii) 0.09 m (viii) 0.083 m (ix) 2 m 67 cm

3. Convert the following into millimetres:
   (i) 7 cm (ii) 4.5 cm (iii) 8 m 63 cm

4. The end-point P of a line-segment PQ is against 1 cm mark and the end-point Q is against the mark indicating 3.7 cm on a ruler. What is the length of the segment PQ?
5. Guess the length and width (and height, if any) of the following objects in centimetres, and then measure them to check how far your guess was correct.
   - (i) A postcard
   - (ii) A match box
   - (iii) Top of teacher's table
   - (iv) A door
6. In Fig. 11.16, verify by measurement the following:
   - (i) AC + BD = AD + BC
   - (ii) AB + CD = AD − BC

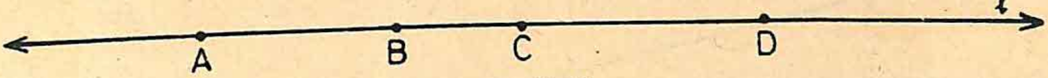


Fig. 11.16

7. Identify the longest and the shortest segments in each of the following figures with the help of a divider and then measure their lengths:

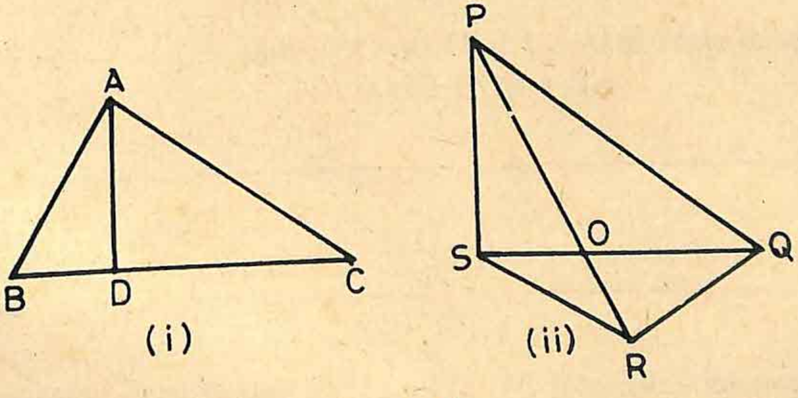


Fig. 11.17

8. In each of the following cases, state whether you can draw segments on the given surfaces:
   - (i) The surface of an egg or apple.
   - (ii) The base of a cone.

9. Construct the line segments whose lengths are:
   (i) 4 cm 7 mm (ii) 12.3 cm (iii) 2.0 cm

10. By measurement, divide a line segment of length 8.4 cm into
    (i) two equal parts (ii) three equal parts

11. In each of the lines given in Fig. 11.18, there are three points A,B,C. By measurement, verify:
    (i) AB + BC = AC
    (ii) AC − BC = AB

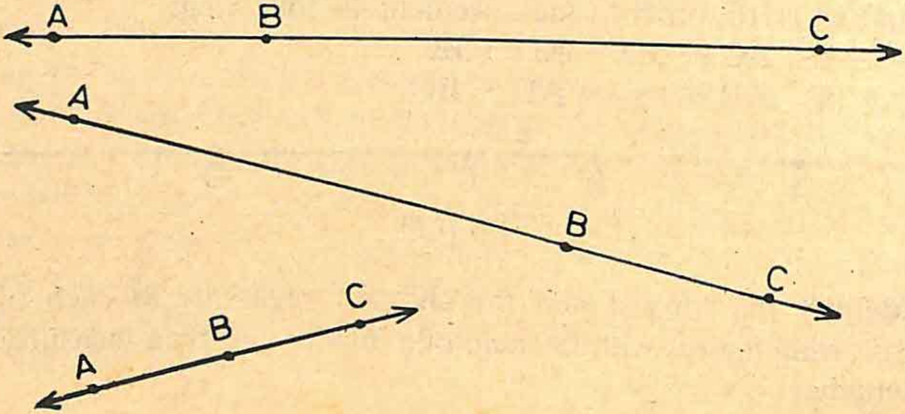


Fig. 11.18

12. In each of the lines in Fig. 11.19, verify that

$$AB + BC + CD = AD$$

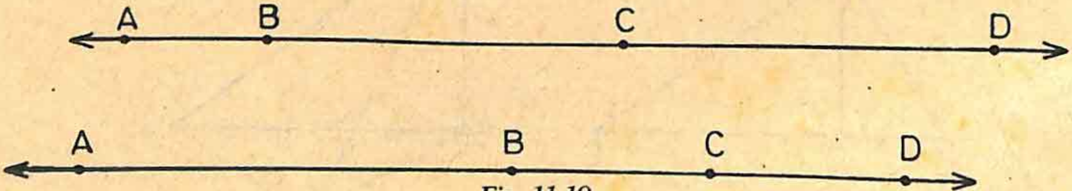


Fig. 11.19

13. Construct a segment AB of length 11 cm in your notebook. From this, cut a segment AC of length 3.7 cm. Measure the remaining segment.

14. Construct two segments of lengths 4.1 cm and 2.5 cm. Construct a segment whose length is equal to the sum of the lengths of these segments.

15. Construct a segment CD whose length is twice the length of segment AB given in Fig. 11.20. (In such a case, we say that CD = 2 AB.)

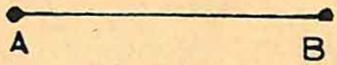


Fig. 11.20

16. If AB = 2.5 cm and CD = 1.5 cm, construct a segment whose length is equal to
    (i) AB − CD (ii) 2 AB (iii) 3 CD
    (iv) AB + CD (v) 2 AB − CD

Things to Remember

1. A line segment is a portion of a line.
2. A line segment has two end-points and a line has none.
3. There is a unique line segment joining two given points A and B. It is denoted by $\overline{AB}$.
4. The length of a line segment AB is denoted by AB.
5. Two or more line segments having the same length are said to be equal.
6. 1 m = 100 cm = 1000 mm, 1 cm = 10 mm,
   1 km = 1000 m, 1 yd = 3ft = 36 in, 1 ft = 12 in,
   1 mile = 1760 yd
7. The distance between two points is the same as the length of the line segment joining these points.

CHAPTER TWELVE

Angles

HERE WE SHALL LEARN another important figure in geometry, which is known as angle. For this, we require some terms like rays. We shall then learn how to measure such angles and how to classify them.

12.1 Ray

Mark a point O on a line l [Fig. 12.1(i)]. By a ray we mean a figure consisting of those points of line l that lie on one side of O including the point O. In Fig. 12.1(i), one ray originating from O on the line is directed towards left.

Some physical examples of rays are a ray of light emitted by the sun [Fig. 12.1(ii)] or a ray of sight [Fig. 12.1(iii)]. The ray of light originates from a point in the sun and extends indefinitely in one direction.

Thus, *the part of a line that extends indefinitely in one direction from a point, say 'O' on a line is called a ray. The point O is called the initial (or end) point of the ray.* A ray has one end-point.

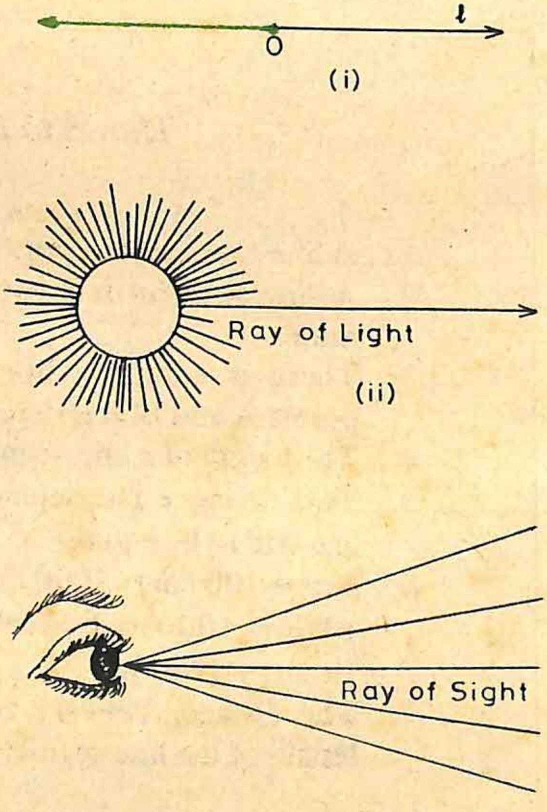


Fig. 12.1: Rays

Experiment 1: Let us mark a point O on a paper. We draw a ray with O as the initial point. Can we draw any other ray which has O as its initial point? How many such rays can you draw? Clearly, we can draw as many rays as we like (Fig. 12.2). Thus, an unlimited number of rays can be drawn with a given point as the initial point.

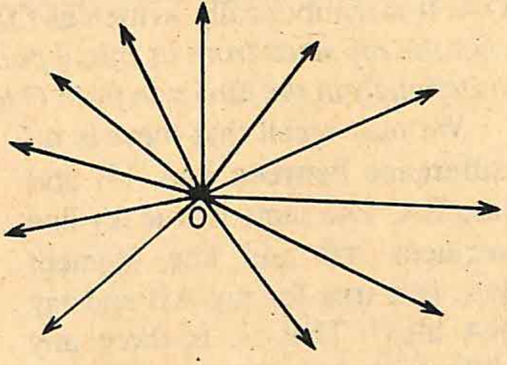


Fig. 12.2: Rays with same initial point

Experiment 2: Let us now see how many rays can be drawn with the given initial point O and passing through a given point A. Join O and A and draw the ray as shown in Fig. 12.3. Can you draw any other ray with O as the initial point and passing through A? If you try, you will find that no other ray is possible to be drawn.

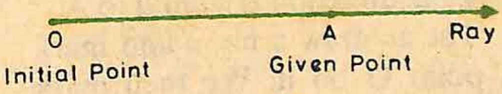


Fig. 12.3

Thus, *there is a unique ray which has O as its initial point and passes through a given point A*.

Let us mark any other point B on the ray drawn in Fig. 12.3 as shown in Fig. 12.4. Is the new ray with O as initial point and passing through B the same as the previous ray with O as initial point and passing through A? Yes.

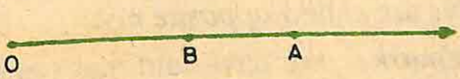


Fig. 12.4

Thus, *a ray is known completely if its initial point and one more point on it are known.*

Since the initial point and one more point of the ray determine the ray uniquely, we name a ray on this basis. Given a ray as shown in Fig. 12.5, we name its

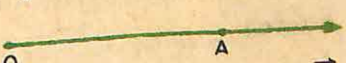


Fig. 12.5: Ray OA or $\overrightarrow{OA}$

initial point as O and mark any other point on it as A. We name it as ray OA. It is symbolically written as OA (read as 'ray $\overrightarrow{OA}$'.). *Ray OA means that the ray starts from its initial point O and passes through A extending indefinitely in the direction from O to A.*

We may recall that there is no difference between line AB and line BA. The same is true for line segment AB and line segment BA. Is it true for ray AB and ray BA also? That is, is there any difference between ray AB and ray BA? Let us mark any two points A and B on a paper and draw the two rays, one with A as initial point and the other with B as initial point [Fig. 12.6]. We can easily see that the two rays are different. Thus, we cannot say that ray AB and ray BA are the same. For ray AB, the direction is from A to B; while for ray BA, the direction is from B to A.

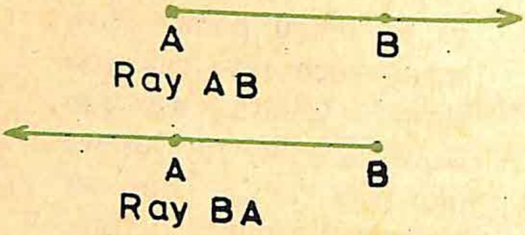


Fig. 12.6

Let us draw a line *l* and mark a point O on it. We then mark two points A and B on this line such that they lie on opposite sides of O as shown in Fig. 12.7. We obtain two rays OA and OB with the same initial point extending indefinitely in opposite directions of the same line. Such rays are called *opposite rays.*

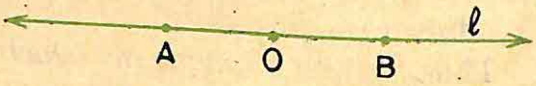
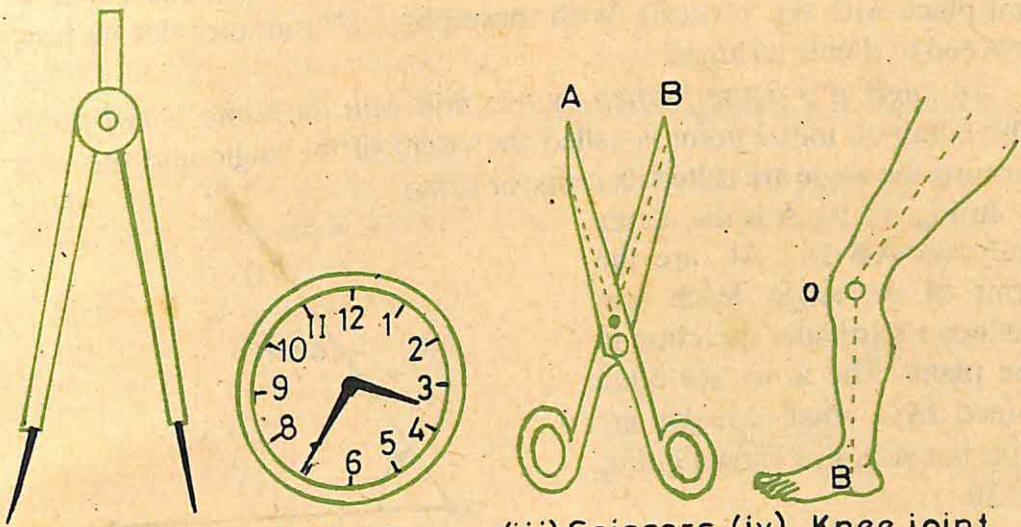


Fig. 12.7: Opposite rays

Remark: We have said that symbols $\overleftrightarrow{AB}$, $\overline{AB}$, AB and $\overrightarrow{AB}$ are used to denote line AB, line segment AB, length of the line segment AB and ray AB, respectively. We feel that the use of these symbols at this stage is unnecessary. Therefore, unless otherwise stated, we shall use the same symbol AB for all the four. It will be clear from the context which of the four we are referring to.

12.2 Angles

You must have come across the physical objects as shown in Fig. 12.8(i) to (iv) which have essentially two arms OA and OB joined together by a hinge. The two arms OA and OB are inclined towards each other and have an opening between them.



(i) Divider (ii) Clock (iii) Scissors (iv) Knee joint

Fig. 12.8: Illustrations of an angle

In fact, we can take these objects in such a way that two rays [two arms of a divider in Fig. 12.8(i), two hands of a clock in Fig. 12.8(ii), two sharp parts of scissors in Fig. 12.8(iii) and the upper part and the lower part of the leg in Fig. 12.8(iv)] are formed with a common initial point (i.e. the joint of the objects mentioned above). This is the idea of an angle in geometry.

The idea of an angle is of great importance in physical phenomenon. For example, the length of the shadow depends on the angle formed by the shadow with the sun ray (in the opposite direction) at the point where the ray meets the ground (Fig. 12.9). The apparent size of an object seen

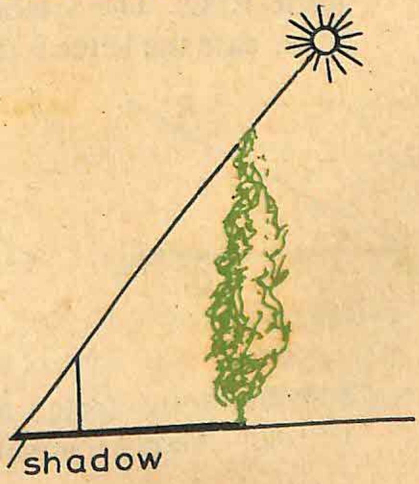


Fig. 12.9

by us depends on the angle made by the object at our eyes; the intensity of heat at a place depends on the inclination of the sun's rays (the angle formed by the ray from the sun that falls on the earth at that place with the vertical). With these physical examples, let us now proceed to define an angle.

An angle is a figure formed by two rays with the same initial point. The common initial point is called the *vertex* of the angle and the rays forming the angle are called its *arms* (or *sides*).

In Fig. 12.10, A is the vertex and rays AB and AC are the arms of the angle. Each arm defines a particular direction in the plane. The arms are often joined by a small circular arc near the vertex as shown in Fig. 12.10.

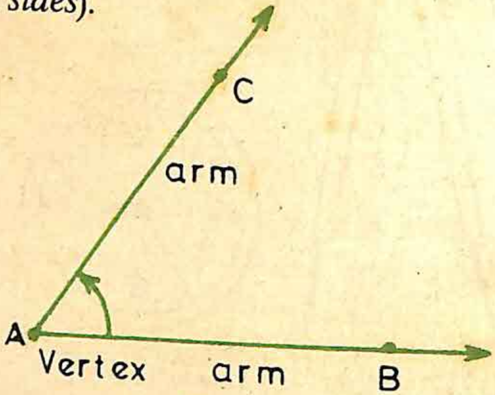
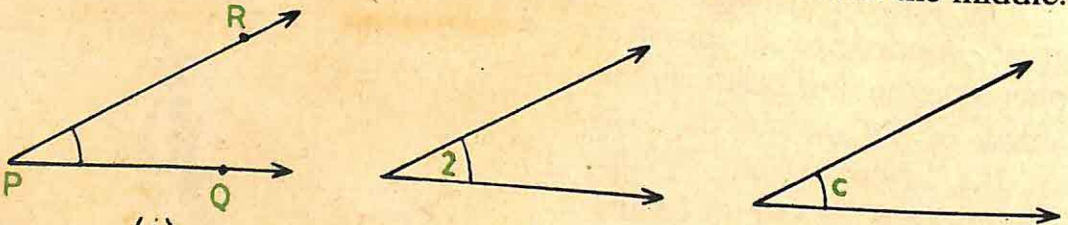


Fig. 12.10

Notation for an Angle: To name a given angle formed by two rays with the same initial point, we name the vertex, say P and name one more point on each arm, say Q and R [Fig. 12.11(i)]. Then the given angle is denoted by ∠QPR or ∠RPQ and read as 'angle QPR' or 'angle RPQ'. The symbol '∠' stands for the angle. Note that in either case the letter P denoting the vertex is written in the middle.



(i) (ii) (iii)

Fig. 12.11: Notation of an angle

Sometimes, we denote an angle simply as ∠P or angle P [Fig. 12.11(i)]. Another notation for an angle is to place a number

(1, 2, 3, etc.) [Fig. 12.11(ii)] or a small letter (a, b, c, etc.) [Fig. 12.11(iii)] near the circular arc. This notation is convenient when several lines are concurrent.

Let us illustrate these notations through an example:

Example: Write all possible names of the angle shown in Fig. 12.12.

Solution: The angle can be denoted as:

$\angle 7$, $\angle A$, $\angle CAQ$, $\angle QAC$, $\angle CAB$, $\angle BAC$, $\angle PAQ$, $\angle QAP$, $\angle PAB$, $\angle BAP$

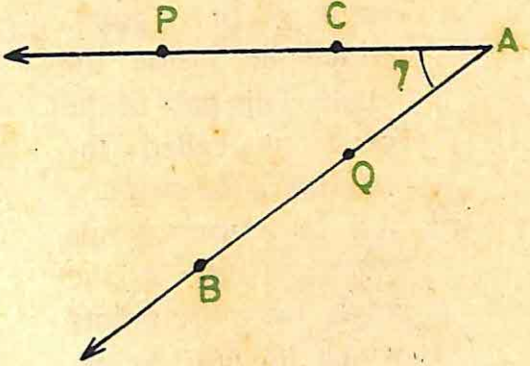
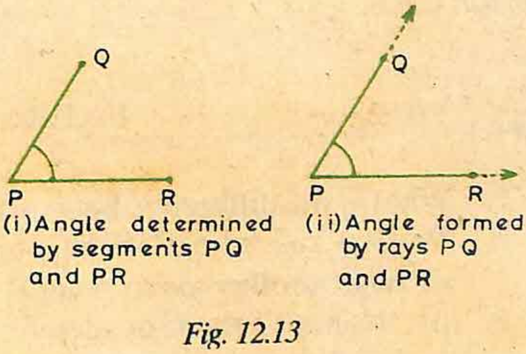


Fig. 12.12

Thus, we observe that if there are other points on the arms of the angle, we can represent the same angle by various names.

Remark: Quite often we come across two segments PQ and PR with a common end-point P as shown in Fig. 12.13(i). We then say that an angle has been determined at P by segments PQ and PR. Thus, *two line segments with a common end-point also determine an angle at that point.* What we really mean by this statement is that the angle is formed by the rays, corresponding to these line segments [Fig. 12.13(ii)]. However, we shall not give much stress on rays and line segments in connection with an angle.



(i) Angle determined by segments PQ and PR

(ii) Angle formed by rays PQ and PR

Fig. 12.13

12.3 Interior and Exterior of an Angle

Let CAB be an angle (Fig. 12.14). All points of the plane of the angle are divided into three parts:

(i) All interior points (such as P) of the angle, i.e. those points which lie inside the angle. This part of the plane is called the *interior of the angle.*

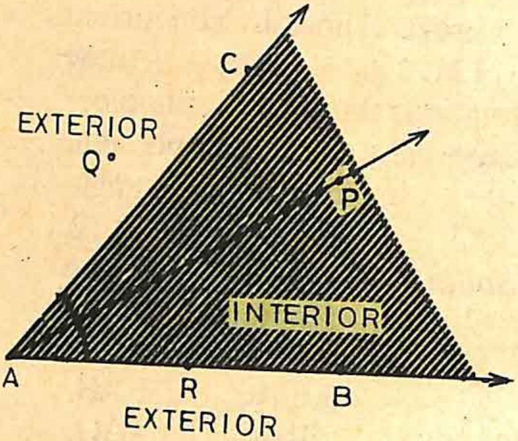


Fig. 12.14

(ii) All exterior points (such as Q) of the angle, i.e. those points which lie outside the angle. This part of the plane is called the *exterior of the angle.*

(iii) All points (such as R) which *lie on the angle itself.*

Further we see that *an angle separates its interior from its exterior.* Note that any path in the plane of an angle from an interior point to an exterior point will pass through some point of the arm AB or AC. The interior of ∠CAB together with the angle itself is called the *angular region* CAB.

EXERCISE 12.1

1. What is the difference between a ray and a line?
2. Draw a ray whose initial point is given at A and which passes through another given point B.
3. (i) Name all the rays shown in Fig. 12.15, whose initial points are A, B and C, respectively.

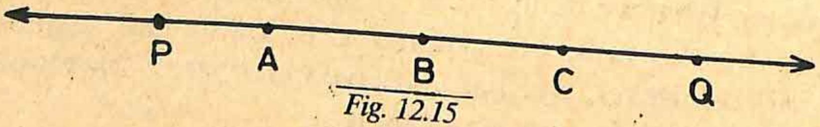


Fig. 12.15

(ii) Is ray AB different from ray AQ?
(iii) Is ray BA different from ray CA?
(iv) Is ray PQ different from ray QP?
(v) Is ray CP different from ray CA?

4. Give three examples of angles from your environment.
5. How many angles are shown in Fig. 12.16? Name them.

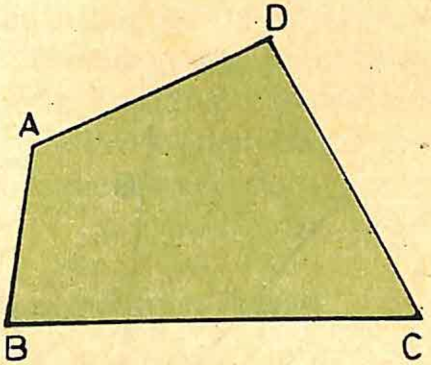


Fig. 12.16

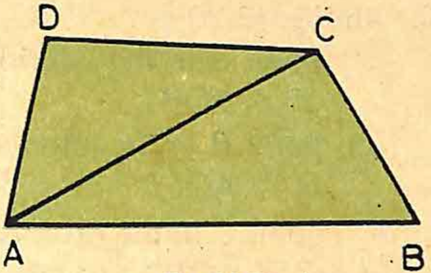


Fig. 12.17

6. How many angles are shown in Fig. 12.17? Name them. *How many of them can be named using the vertex letter only?*
7. From Fig. 12.18, list the points which are (i) in the interior of $\angle P$ (ii) in the exterior of $\angle P$ and (iii) lie on $\angle P$.
8. Write the arms and the vertex of the angle LMP.

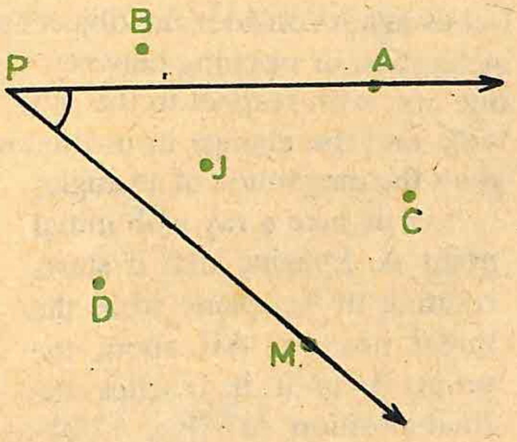


Fig. 12.18

9. In Fig. 12.19, write another name for
   (i) ∠ 1
   (ii) ∠ 2
   (iii) ∠ 3
   (iv) ∠ 4

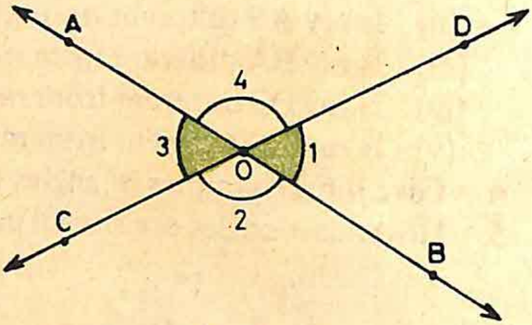


Fig. 12.19

10. In Fig. 12.20, is
   (i) point B in the interior of ∠ AOB?
   (ii) point B in the interior of ∠ AOC?
   (iii) point C in the exterior of ∠ AOB?
   (iv) point D in the exterior of ∠ AOC?
   (v) point A in the interior of ∠ AOD?

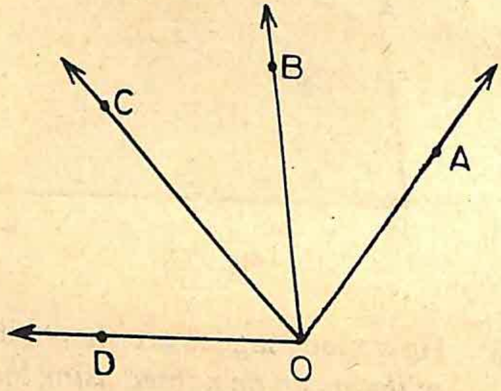


Fig. 12.20

12.4 Magnitude of an Angle

Let us again consider the objects as shown in Fig. 12.8. Observe that the inclination or opening between OA and OB can be changed by rotating one arm with respect to the other; the opening may be made narrow or wide and the change in inclination (or opening) between the two arms gives the magnitude of an angle.

Let us take a ray with initial point A. Imagine that it starts rotating in the plane from the initial position AB about the point A until it reaches the final position AC (Fig. 12.21).

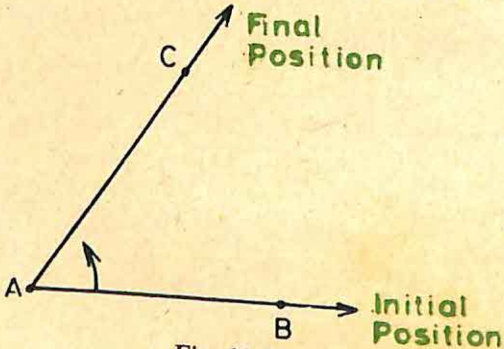


Fig. 12.21

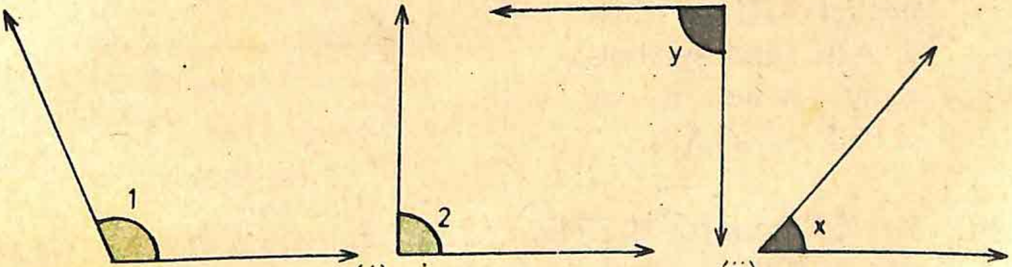
We may say that an angle has been described by the rotating ray with A as vertex and its arms as AB and AC.

Thus we see that the inclination between two arms (rays) forming an angle can be increased or decreased. In other words, two angles may have different inclinations between their arms. Thus, we can say that these two angles have different magnitudes (or measures).

By the *magnitude* of an angle we, therefore, mean the amount of rotation through which one arm must be rotated about the vertex to bring it to the position of the other arm. If the magnitude of one angle is greater than that of the other, we say that the first angle is greater (or larger) than the second one.

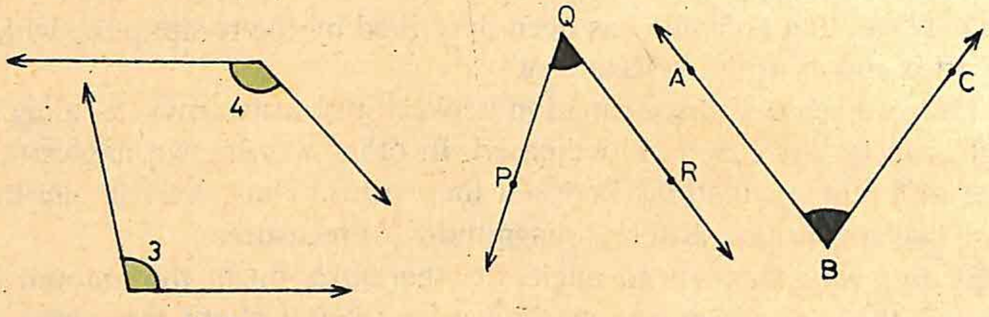
12.5 Comparison of Angles

It becomes often necessary to know, between two angles, which one is greater or less than or equal to the other. This is what we call comparison of two angles. To do this, we need to know what are called 'magnitudes of angles' and the (order) relation between them. When the magnitudes of two angles differ very much, by simply looking at them, we can say which angle is greater and which is smaller. For example, for the pairs of angles given in Fig. 12.22, we can easily say that $\angle 1$ is greater than $\angle 2$, $\angle x$ is smaller (less) than $\angle y$, $\angle 3$ is smaller than $\angle 4$ and $\angle ABC$ is greater than $\angle PQR$.



(i) (ii)

Fig. 12.22



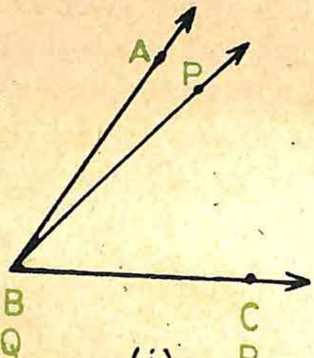
(iii) (iv)

Fig. 12.22

It is not always possible to compare angles by observation. A better method is to compare them by tracing one angle and superimposing the traced angle on the other. Suppose we have to compare two angles ABC and PQR by superimposition. We proceed as follows:

We place a tracing paper on one of the angles, say ∠PQR and copy this angle on the tracing paper. Next, this traced angle PQR is placed over angle ABC such that point Q is on point B and ray QR is along ray BC. What can be the possible positions of the other ray (arm) QP of ∠PQR? There are following three possibilities for the ray QP:

(i) Ray QP may be between rays BC and BA [Fig. 12.23(i)]. In such a situation, we say that ∠PQR is smaller (less) than ∠ABC and symbolically write it as ∠PQR < ∠ABC.



(i)

Fig. 12.23

(ii) Ray QP is on ray BA [Fig. 12.23(ii)]. In such a situation, we say that ∠PQR is equal to ∠ABC and symbolically write it as ∠PQR = ∠ABC.

Thus, *two or more angles having the same magnitude (measure) are said to be equal.*

(iii) Ray QP is beyond ray BA [(Fig. 12.23(iii)]. In such a situation, we say that ∠PQR is greater than ∠ABC and symbolically write it as ∠PQR > ABC.

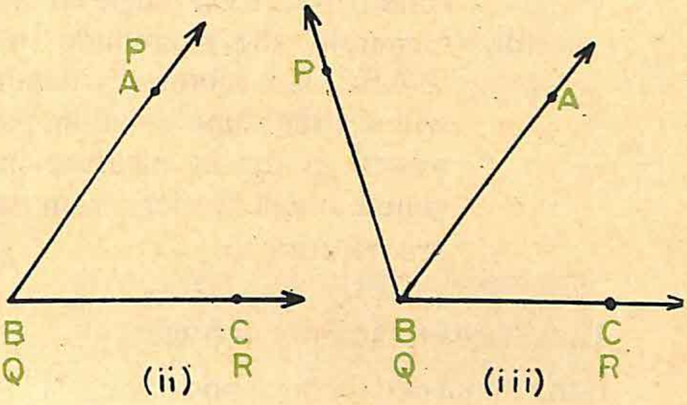


Fig. 12.23

Suggested Activities: (i) On a piece of paper draw two angles and compare them using a tracing paper.

(ii) Fold two sheets of paper by making intersecting creases and compare the angles so formed.

All these methods of comparing angles are not complete for we do not know what each angle stands for in terms of numbers. A better way is to measure the two angles in terms of some standard angle (called a unit angle) and compare their measures (magnitudes).

Remarks: (i) We can imagine as well about an angle due to rotation of a ray from the position AC to AB, with A as an initial point.

(ii) Two rays PR and PQ form two angles at P as shown in Fig. 12.24. Both of them can be denoted as ∠RPQ. In general, one of them is greater than the other. Therefore, to avoid ambiguity, unless otherwise

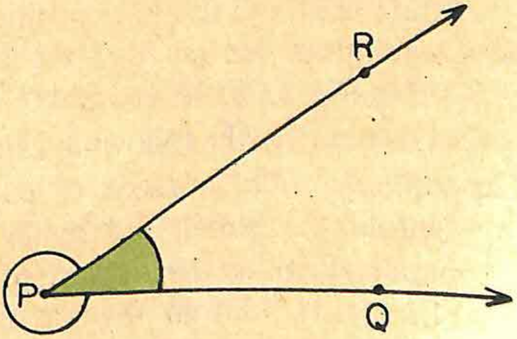


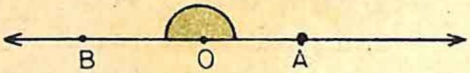
Fig. 12.24

stated, by $\angle$RPQ (angle RPQ) we shall mean the smaller one.

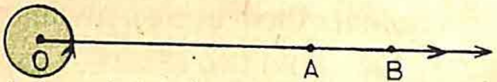
(iii) Generally, the 'magnitude' or the 'measure' of an angle, say $\angle$ABC, is symbolically denoted as $m\angle$ABC. However, we will use the same notation, namely '$\angle$ABC', for angle ABC as well as for its measure or magnitude, unless otherwise stated. It will be clear from the context, which of the two we are referring to.

12.6 Degree Measure of Angles

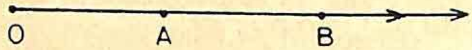
If the initial and the final positions of a rotating ray are opposite to each other (on the same line), the angle formed is called a *straight angle.* In other words, *an angle formed by two opposite rays is called a straight angle.* In Fig. 12.25(i), the angle formed by two opposite rays OA and OB is a straight angle.



(i) Straight angle



(ii) Complete angle



(iii) Zero angle

Fig. 12.25

If a rotating ray, after making a complete revolution, coincides with the initial position, the angle formed is called a *complete angle* as shown in Fig. 12.25(ii). Here the arms OA and OB coincide after making a complete revolution.

If the initial and the final positions of a ray coincide without making any revolution, we say that the angle formed is a *zero angle.* Thus, $\angle$AOB in Fig. 12.25(iii) is a zero angle.

Let us perform the following experiment:

Experiment: Fold a piece of paper to make a straight crease AB. Unfold the paper and make another crease CD by folding the paper again so that a part of the straight crease AB falls on the other part. Unfold the paper. We observe that two intersecting creases AB and CD make four angles at the point of intersection O

(Fig. 12.26). Trace one of these and place on the other three angles. You will find that all the four angles are equal. Each one of them is called a *right angle.* The angle at the corner of this paper, a blackboard or a room are all examples of right angles.

We could adopt a right angle as our unit for measuring angles. If we do so, then from Fig. 12.26, we can easily observe that
a complete angle = 4 right angles
a straight angle = 2 right angles.
But for most purposes, it is too big a unit as many of the angles would then have to be measured as fraction of a right angle. It is, therefore, convenient to take a fraction of a right angle itself as the basic unit of measurement for angles. We divide a right angle into 90 equal parts. Each part is called a *degree* and we adopt this as our basic unit of measurement for an angle. The ‘degree’ is denoted by a small circle ‘°’, written as a superscript. Thus, 1 degree, 0 degree and 55 degree are written as 1°, 0° and 55°, respectively.

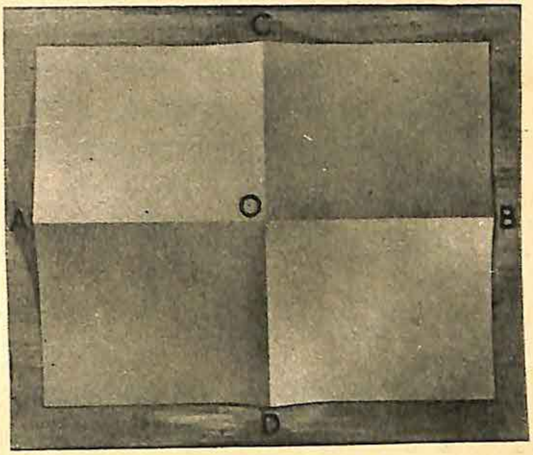


Fig. 12.26

The degree is further divided into *sixty minutes* and each minute into *sixty seconds.* Minutes and seconds are denoted by one prime (′) and two primes (″), respectively, each written as a superscript. Thus, 2 minutes is denoted as 2′, 3 seconds is denoted as 3″, etc. Minutes and seconds are used when very accurate measurements of angles are required, for example, in astronomy and physics. However, we will construct or measure angles correct to a degree only. The above relations can be summarized as follows:

EXERCISE 12.2

1. By simply looking at the pairs of angles given in Fig. 12.29, state which of the angles in each of the pairs is greater:

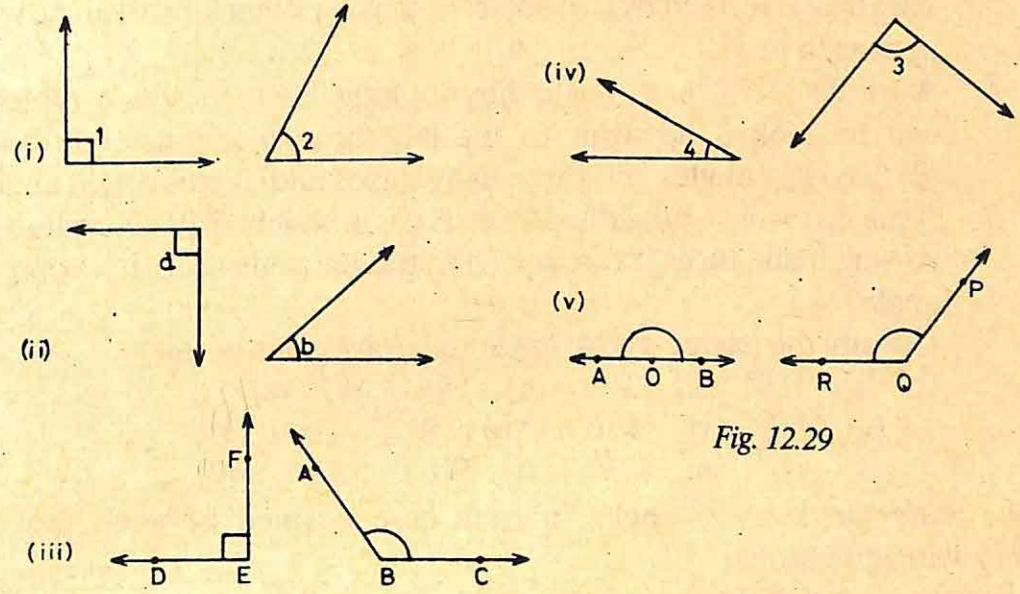


Fig. 12.29

2. Some of the pairs of angles are given in Fig. 12.30. Compare each pair using a tracing paper and answer the following:
   - (a) In which pairs the angles are equal?
   - (b) Is ∠a < ∠b?
   - (c) Is ∠4 > ∠ 3?
   - (d) Is ∠ABC > ∠PQR?
   - (e) Is ∠1 < ∠2?
   - (f) Is ∠ABC > ∠DEF ?

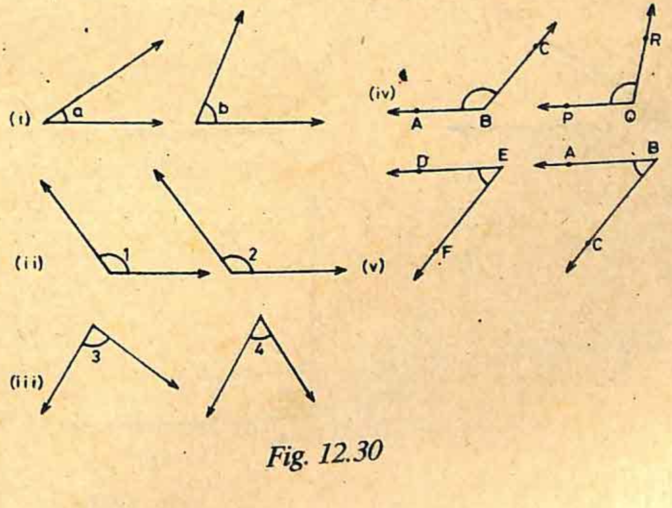


Fig. 12.30

Zero angle = 0°
1 right angle = 90°
1 straight angle = 2 right angles = 180°
1 complete angle = 4 right angles = 360°
1° = 60′
1′ = 60″

Remark : There is also a system of measuring angles, where a right angle is divided into 100 equal parts. Each of these parts is called a *grade.* One grade is denoted as 1^g. We can call it the metric system of measuring angles. But degree as a unit of measuring angles is widespread and well established. Therefore, we will not use this grade measure in our book.

12.7 Kinds of Angles

You have already learnt about a zero angle, right angle, straight angle and complete angle. Now we discuss some other kinds of angles.

(i) *Acute Angle*

An angle which is greater than a zero angle but less than a right angle is called an *acute angle* (Fig. 12.27). Its measure is greater than 0° and less than 90°.

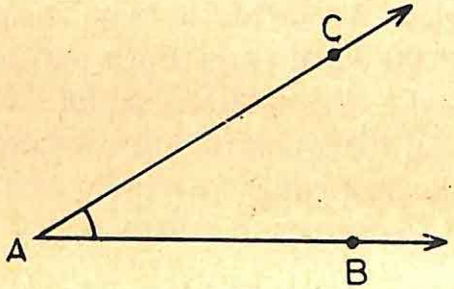


Fig. 12.27

(*ii*) *Obtuse Angle*

An angle which is greater than a right angle but less than a straight angle is called an *obtuse angle* (Fig. 12.28). Its measure is greater than 90° but less than 180°.

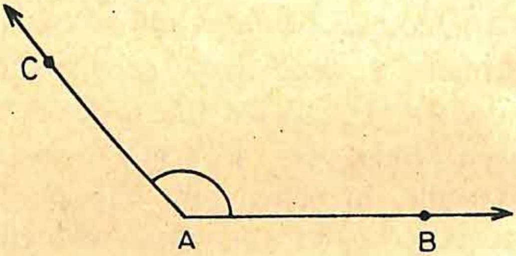


Fig. 12.28

3. Give two examples each of right, acute and obtuse angles from your environment.
4 Using your upper arm and fore arm form acute, right and obtuse angles at the elbow joint.
5. An angle is formed by two adjacent fingers. What kind of angle will it appear to be?
6. A traffic policeman is standing looking East. In which direction will he look if he turns to the left through (i) one right angle, (ii) two right angles, (iii) three right angles and (iv) four right angles?
7. Asha is rowing a boat due North-East. In which direction will she be rowing if she turns it through (i) a straight angle and (ii) a complete angle?
8. Classify the angles whose magnitudes are given below:

   (i) 118° (ii) 19° (iii) 155° (iv) 0°
   (v) 70° (vi) 180° (vii) 89.5° (viii) 30°
   (ix) 90° (x) 179° (xi) 90.5° (xii) 360°

9. State the kind of angle, in each case, formed between the following directions:

   (i) East and West (ii) East and North
   (iii) North and North-East (iv) North and South-East

10. State the kind of each of the following angles:

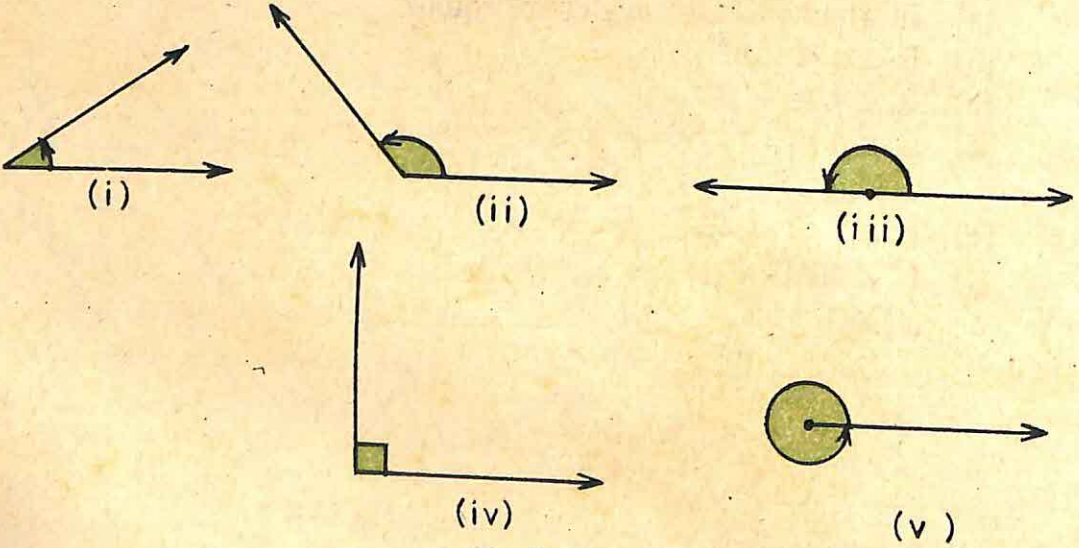

(i) (ii) (iii) (iv) (v)

Fig. 12.31

11. Using only a ruler, draw an acute angle, a right angle and an obtuse angle in your notebook and name them.
12. By folding the paper in different ways, form a right angle, an obtuse angle and an acute angle.

12.8 Pairs of Angles

We often come across pairs of angles which have been given specific names.

(i) Adjacent Angles

Look at Fig. 12.32. We have two angles BAC and CAD with same vertex A and the arm AC common to them. The two arms AB and AD are opposite to (the line containing) the common arm AC. Two such angles with same vertex, one arm common and with their other arms lying on opposite sides of this common arm are called *adjacent angles.*

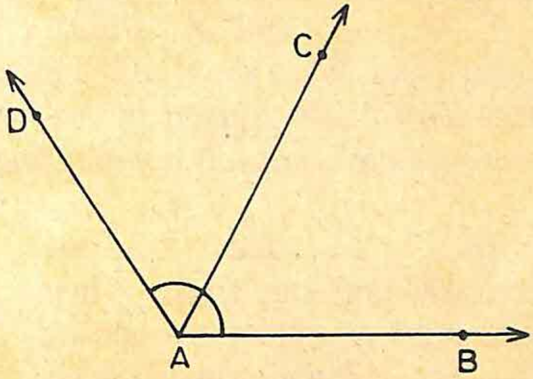


Fig. 12.32: Adjacent angles

Alternatively, we may say that *two angles in a plane are adjacent angles if they have a common vertex, a common arm and their interiors do not overlap.* Thus, in the above figure, ∠BAC and ∠CAD are adjacent angles. Are ∠BAC and ∠BAD adjacent? No. (Why?)

(ii) Linear Pair

Look at Fig. 12.33. Here ∠BAC and ∠CAD are adjacent angles. Observe that their two non-common arms AB and AD are opposite rays. Such a pair of adjacent angles is called a *linear pair.*

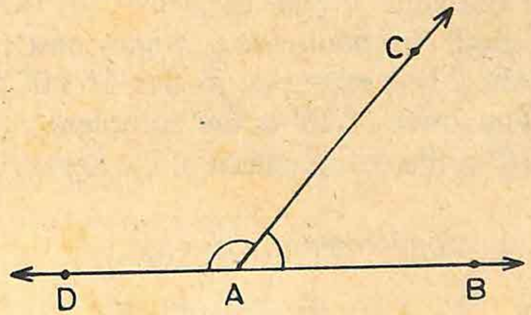


Fig. 12.33: Linear Pair

(iii) Vertically Opposite Angles

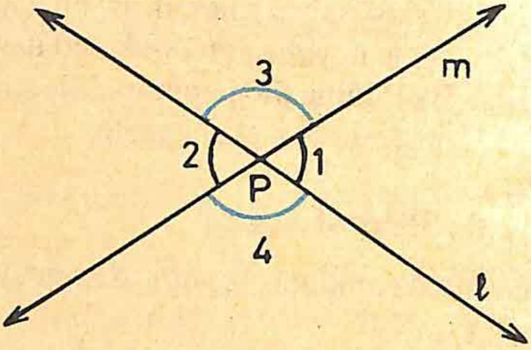


Fig. 12.34: Vertically opposite angles

Look at Fig. 12.34 in which two lines *l* and m are intersecting at a point P. We can see that with the intersection of these two lines, four angles have been formed. We can also observe that ∠1 and ∠3 form a linear pair. Similarly, ∠3 and ∠2, ∠2 and ∠4, ∠1 and ∠4 form linear pairs. What about ∠1 and ∠2? These angles have no common arm. *Two angles formed by two intersecting lines having no common arm are called vertically opposite angles.*

Thus, ∠1 and ∠2 are vertically opposite angles. Their arms form two pairs of opposite rays. Are ∠3 and ∠4 vertically opposite angles? Yes. Are ∠a and ∠c in Fig. 12.35 vertically opposite angles? No. (Why?)

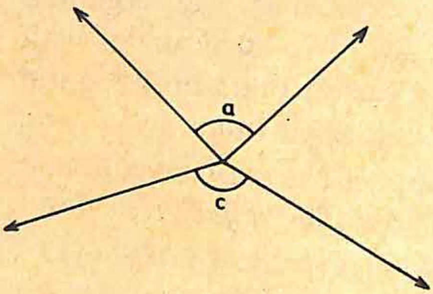


Fig. 12.35

(iv) Complementary Angles

If the sum of (the measures of) two angles is 90°, then the angles are called *complementary angles* and each is called a *complement* of the other. For example, angles of 20° and 70° are complementary angles. The angle of 20° is the complement of the angle of 70° and the angle of 70° is the complement of the angle of 20°

(v) Supplementary Angles

If the sum of two angles is 180°, then the angles are called *supplementary angles* and each of them is called the *supplement* of the other. For example, angles of 50° and 130° are supplementary angles. The angle of 50° is the supplement of the angle of 130° and the angle of 130° is the supplement of the angle of 50°.

Note that the *angles of a linear pair are supplementary.* Is it true that supplementary angles always form a linear pair? No (See Fig. 12.36). ∠ABC + ∠DEF = 110° + 70° = 180°. But they are not adjacent angles, and hence do not form a linear pair.

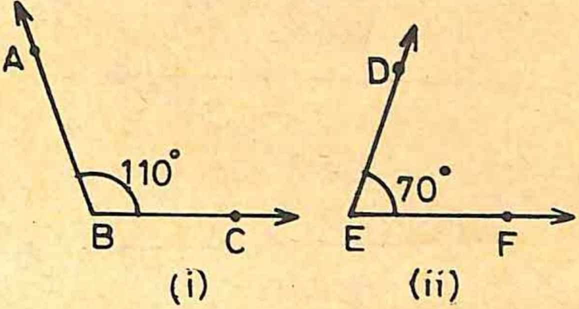


Fig. 12.36

EXERCISE 12.3

1. Write down each pair of adjacent angles shown in Fig. 12.37.
2. In Fig. 12.37,
   - (i) are ∠AOB and ∠BOC adjacent angles?
   - (ii) are ∠BOC and ∠COD adjacent angles?
   - (iii) are ∠AOB and ∠COD adjacent angles?
   - (iv) are ∠AOB and ∠AOC adjacent angles?

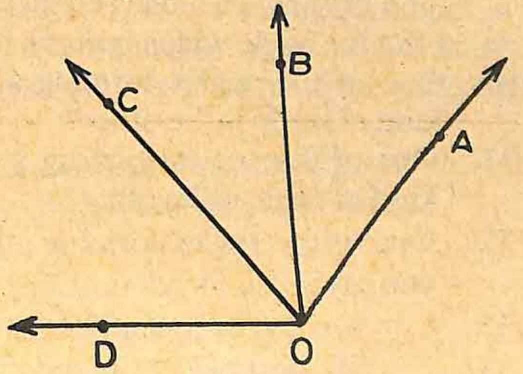


Fig. 12.37

3. From Fig. 12.38, write down
   - (i) each linear pair.
   - (ii) each pair of vertically opposite angles.

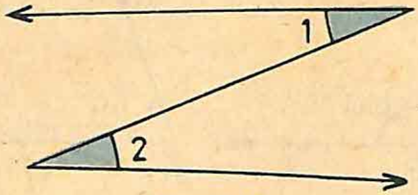


Fig. 12.39

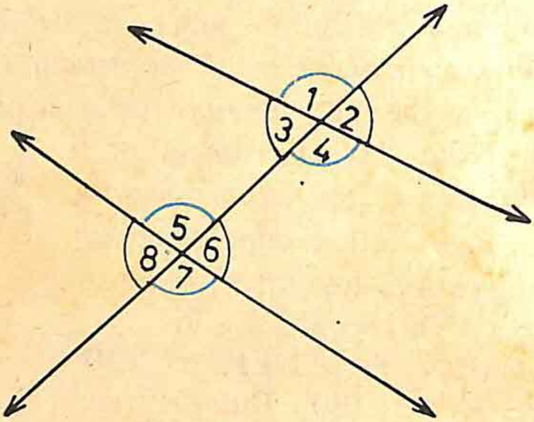


Fig. 12.38

4. Are the angles 1 and 2 given in Fig. 12.39 adjacent angles?
5. Find the supplement of each of the following angles:
   (i) 70° (ii) 135° (iii) 50° (iv) 120° (v) 90°
6. Find the complement of each of the following angles:
   (i) 55° (ii) 73° (iii) 45° (iv) 40° (v) 30°
7. Identify which of the following pairs of angles are complementary and which are supplementary:
   (i) 60°, 30° (ii) 160°, 20° (iii) 73°, 17°
   (iv) 80°, 10° (v) 42°, 138° (vi) 90°, 90°
   (vii) 45°, 45°
8. Find the angle which is equal to its supplement.
9. Find the angle which is equal to its complement.
10. One of the angles forming a linear pair is an acute angle. What kind of angle is the other?
11. One of the angles forming a linear pair is an obtuse angle. What kind of angle is the other?
12. One of the angles forming a linear pair is a right angle. What can you say about its other angle?
13. Can two angles be supplementary, if *both of them* be
    (i) obtuse? (ii) right? (iii) acute?
14. Can two acute angles form a linear pair?
15. An angle is greater than 45°. Is its complement greater than or equal to or less than 45°?

16. In Fig. 12.40, O lies on line AC.
    (i) Are ∠COD and ∠AOD forming a linear pair?
    (ii) Are ∠1 and ∠2 vertically opposite angles?
    (iii) Are angles AOB and BOC supplementary?

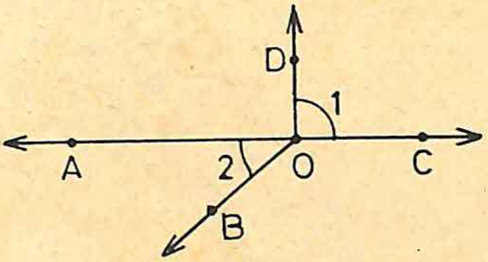


Fig. 12.40

17. In Fig. 12.41, ∠BAC is a right angle.
    (i) Can you say that ∠BAD and ∠CAD are complementary angles?
    (ii) Can you say that ∠BAD and ∠CAD are adjacent angles?

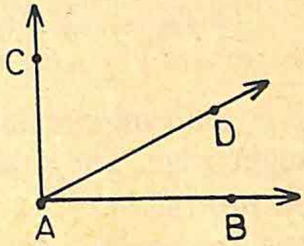


Fig. 12.41

18. Which of the following statements are true (T) and which are false (F)?
    (i) Adjacent angles can be complementary.
    (ii) Adjacent supplementary angles form a linear pair.
    (iii) Two supplementary angles always form a linear pair.
    (iv) If two lines intersect, then one pair of vertically opposite angles always consists of acute angles and the other obtuse angles.
    (v) Two adjacent angles are always supplementary.
    (vi) Angles of a linear pair are always supplementary.

12.9 Protractor

A protractor is an instrument used for measuring a given angle and constructing an angle of given magnitude (Fig. 12.42). It is semi-circular in shape and is usually made of plastic or metal. It has degree marks on

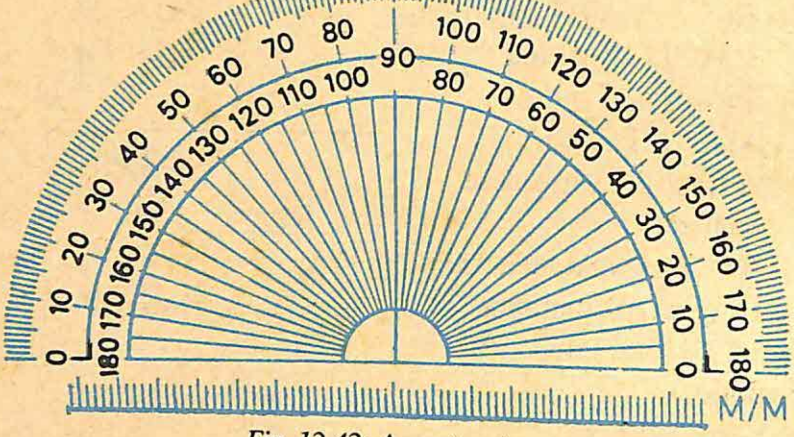


Fig. 12.42: A protractor

the curved edge (semi-circular arc) and a 0 − 180 line along or parallel to the straight edge. The mid-point of this 0 − 180 line is called the *centre* of the protractor.

The curved edge is divided into 180 equal parts by small radial marks so that the angle formed by two rays with common initial point as the centre of the semicircle and passing through two consecutive marks is 1/180 of a straight angle, i.e. 1°.

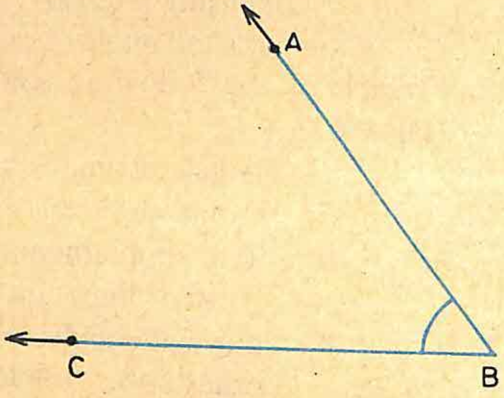


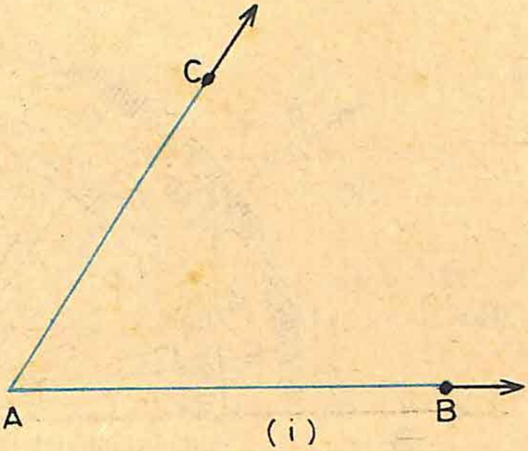
Fig.12.43

The marks are usually written along the curved edge at an interval of 10° starting from 0 at the one end to 180 at the other. The marks are also written in the reverse order for convenience in measuring angles such as ∠ABC given in Fig. 12.43. For this reason, 0 − 180 line can also be named as 0 − 0 line.

(a) To measure a given angle

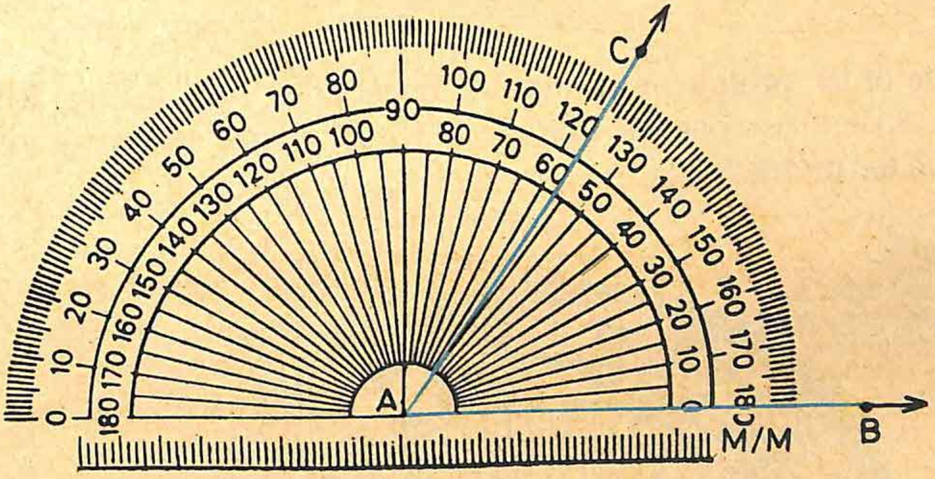
Let us measure a given angle with the help of a protractor.

Suppose ∠BAC is given [Fig. 12.44(i)]. Place the protractor on the angle such that the centre of the protractor is on the vertex of the angle and 0 – 180 line is along one of the arms, say AB of the angle. Read off the degree mark on the curved edge of the protractor through which the other arm AC passes. This should be read on the scale which has the 0 mark on the first arm. In Fig. 12.44(ii), the measure of ∠BAC is seen to be 57° and we write ∠BAC = 57°.



(i)

Fig. 12.44



(ii)

Fig. 12.44

Remark: If the given angle is as shown in Fig. 12.45, then the angle should be read from the other markings. Here, ∠BAC = 35°.

(b) *To construct an angle of given magnitude*

Here we will learn how to construct an angle of given magnitude, say 48°. Draw a ray AB (Fig. 12.46) and place the protractor such that the

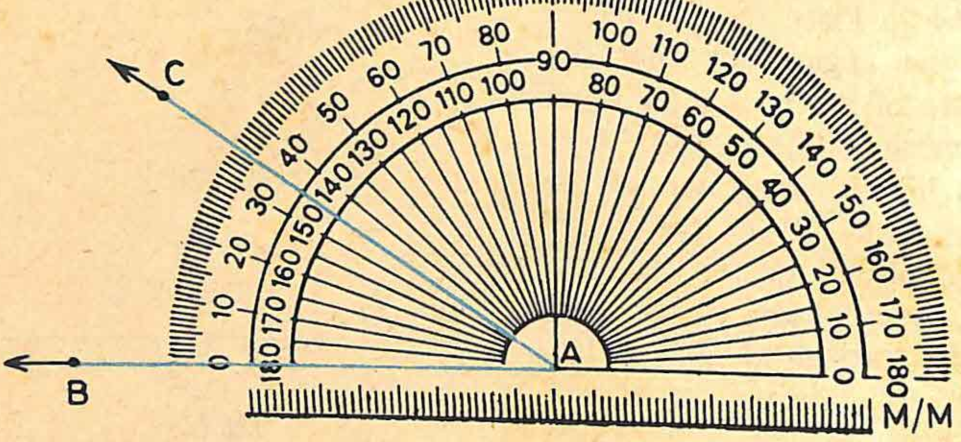


Fig. 12.45

centre of the protractor falls on the initial point A of the ray AB and 0 − 180 line lies along AB. Mark point C on the paper against the mark of 48° on the protractor.

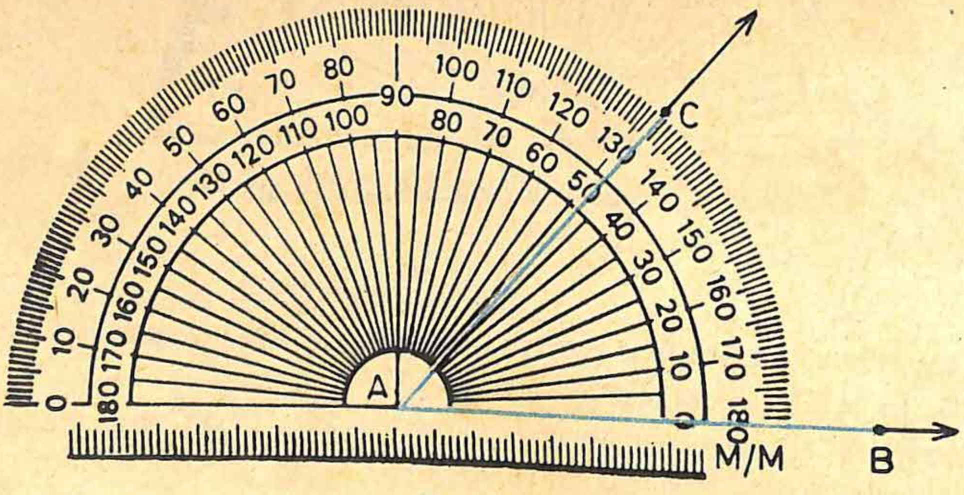


Fig. 12.46

Remove the protractor and draw ray AC. Then, ∠BAC is the required angle, i.e. ∠BAC = 48°.

EXERCISE 12.4

1. What is the magnitude of the angle formed by the hands of a clock when it is (i) 2 O' clock? (ii) 5 O' clock?
2. Construct the following angles with the help of your protractor:
   (i) 43° (ii) 35° (iii) 110° (iv) 90° (v) 180°
   (vi) 21° (vii) 165° (viii) 15° (ix) 65°.
3. Construct a right angle ABC with the help of a protractor. Mark a point D in the interior of ∠ABC. Draw ray BD. Measure ∠ABD and ∠DBC. Verify that they are complementary.
4. Draw two intersecting lines AB and CD as shown in Fig. 12.47. Measure ∠AOD and ∠BOC. Are they equal? Measure ∠AOC and ∠BOD. Are they equal? What conclusion can you draw from it?

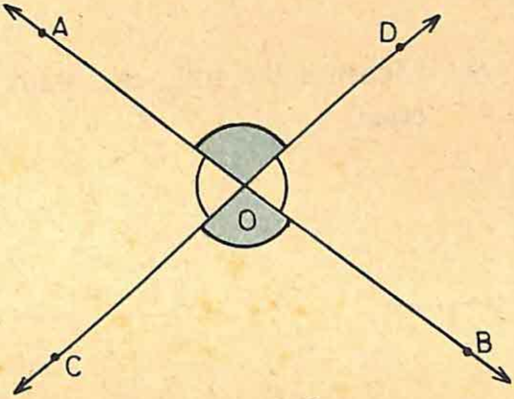


Fig. 12.47

5. Draw a line DB and take a point A on it between D and B. Draw any ray AC. Verify by measurement that ∠BAC and ∠CAD are supplementary.
6. Draw any ∠BAC and take a ray AD in the interior of this angle. Verify by measurement that ∠BAC is equal to the sum of the angles BAD and DAC.
7. Measure each of the pairs of angles given in Fig. 12.48 and determine which of them is smaller.

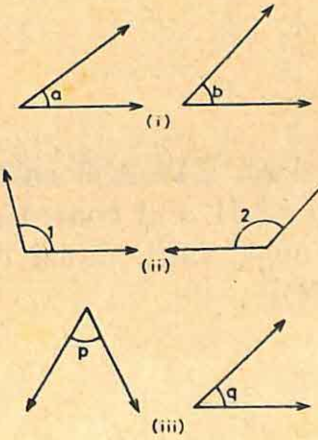


Fig. 12.48

8. Measure each of the following angles and classify it as acute, obtuse or right:

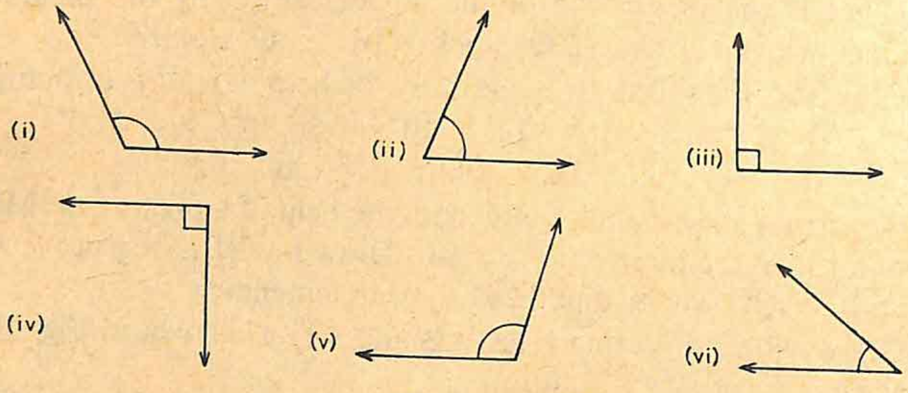


Fig. 12.49

9. Measure the following pairs of angles and state which pairs are equal:

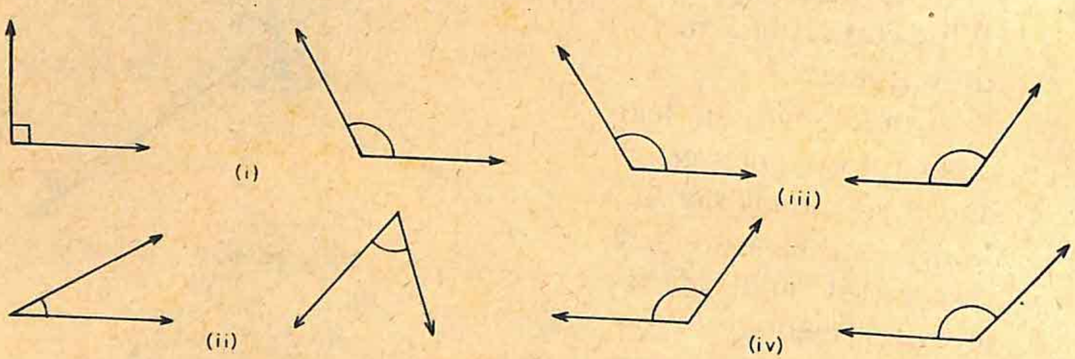


Fig. 12.50

10. Measure ∠PQR given in Fig. 12.51 and construct an angle DEF equal to ∠PQR.

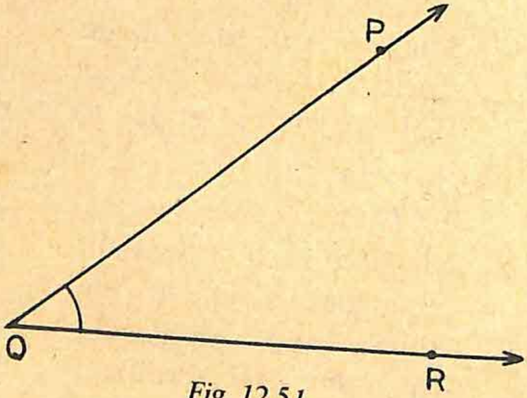


Fig. 12.51

12.10 Set-squares

In the geometrical box, there are two triangular shaped instruments called *set-squares* (Fig. 12.52). In one, the angles are 30°, 90°, 60° and in the other the angles are 90°, 45°, 45°. Set-squares are usually made of transparent plastic, wood or metal and are a few millimetres thick. Sometimes the two perpendicular edges are graduated, one in centimetres and the other in inches. These set-squares can also be used to measure line segments. We may call the set-square in Fig. 12.52(i) as the 30°set-square and the set-square in 12.52(ii) as the 45° set-square.

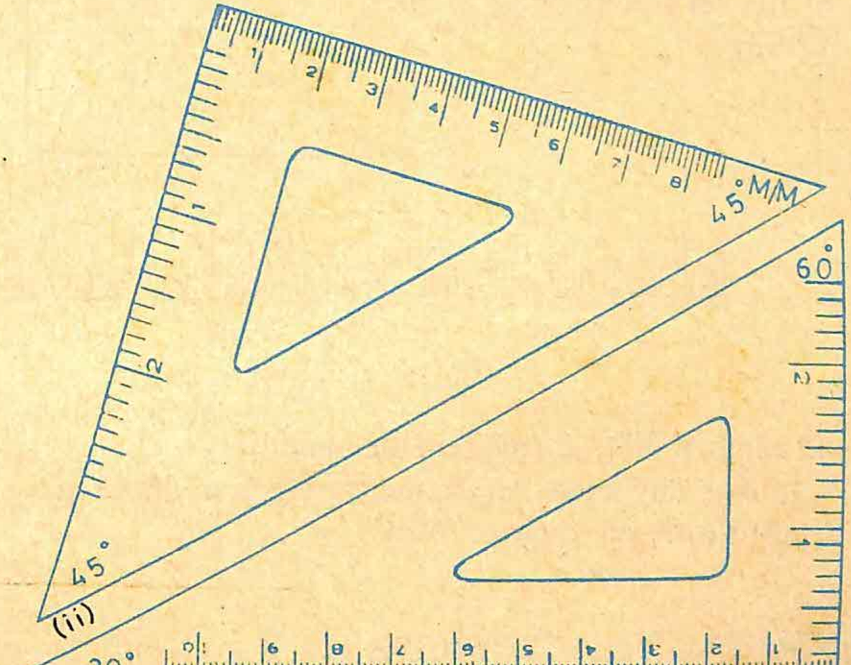


Fig. 12.52

Set-squares have several uses. These can be used to construct angles of 30°, 45°, 60° and 90° in addition to drawing parallel and perpendicular lines. To construct an angle of 30°, we go through the following steps, as shown in Fig. 12.53.

Step 1: Place 30° set-square on a sheet of paper.

Step 2: Hold the set-square down firmly with one hand and with other, draw two rays AB and AC along the edges using a sharp pointed pencil from the vertex of 30° angle of the set-square (Fig. 12.53).

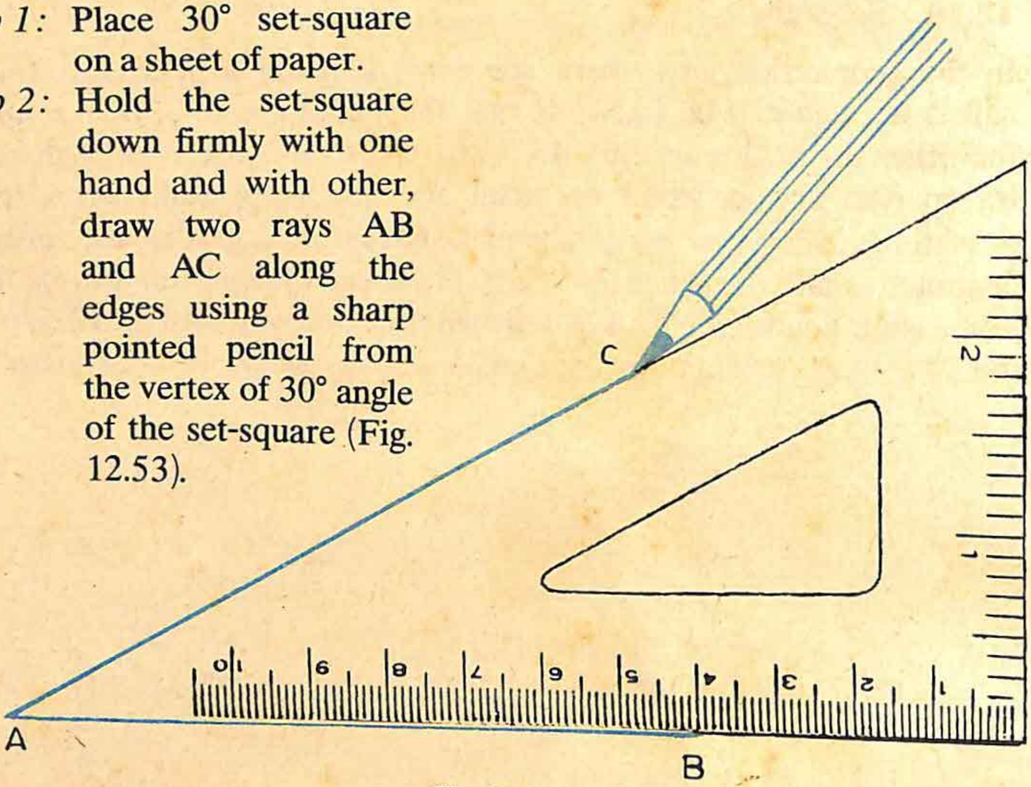


Fig. 12.53

Thus, an angle of 30° is formed on the paper.

Remark: In fact, any angle whose measure is a multiple of 15° can be constructed with set-squares. (Why?)

EXERCISE 12.5

1. Construct the following angles using set-squares:
   (i) 45° (ii) 90° (iii) 60° (iv) 105° (v) 75° (vi) 150°
   [*Hint:* 60° + 45° = 105°]
2. Given a line BC and a point A on it, construct a ray AD using set-squares so that ∠DAC is
   (i) 30° (ii) 150°
   [*Hint:* Place the set-square such that its one vertex coincides with A.]

12.11 Perpendicular Lines and their Construction

Two lines are said to be perpendicular to each other or simply perpendicular if one of the angles formed by them is a right angle (90°). They are also said to be *at right angles to each other* or simply *at right angles.* The symbol '⊥' is used to denote perpendicularity and is read as 'is perpendicular to'. In Fig. 12.54(i), lines l and m are perpendicular and we write line l ⊥ line m and read 'line l is perpendicular to line m'.

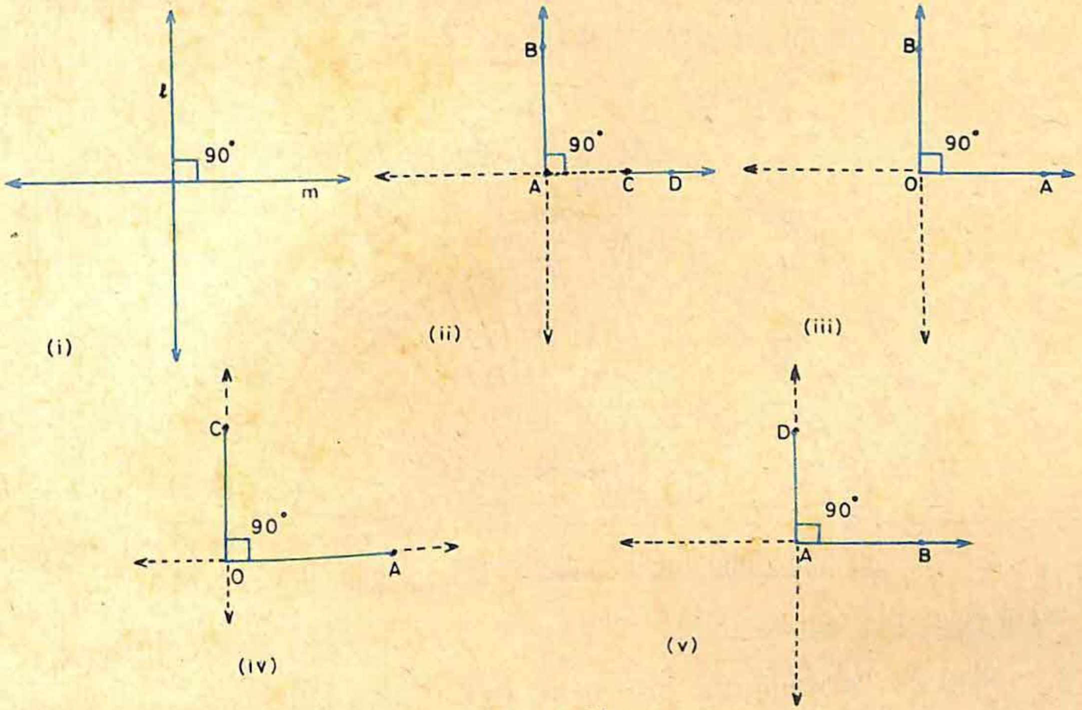


Fig. 12.54

Remark: We also say that two rays are perpendicular if the corresponding lines determined by them are perpendicular. Thus, in Fig. 12.54 (iii), ray OA ⊥ ray OB and in Fig. 12.54(ii), ray AB ⊥ ray CD. Similarly, we say that two segments or a ray and a segment are perpendicular if the two corresponding lines determined by them are perpendicular, e.g. in Fig. 12.54(iv), segment OA ⊥ segment OC and in Fig. 12.54(v), ray AB ⊥ segment AD.

We shall now study some methods of constructing perpendicular lines.

(i) *Construction of a line perpendicular to a given line l at a given point A on it*

(a) *Using a protractor:* For this construction, we go through the following steps:

Step 1: Place the protractor on the paper with its centre coinciding with given point A and its 0 − 180 line lying along the given line *l* (Fig. 12.55).

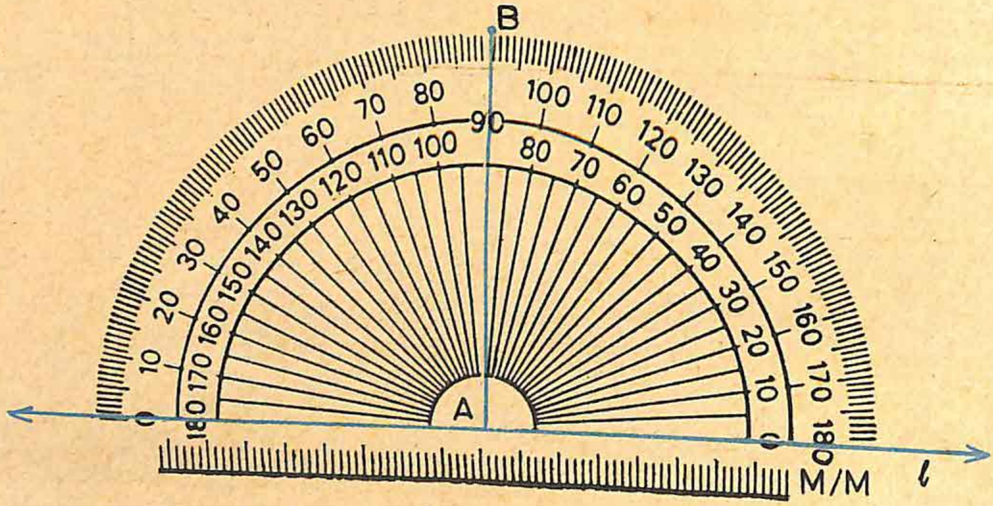


Fig. 12.55

Step 2: Holding the protractor fixed, mark with a pencil a point B on the paper against the 90° mark of the protractor.

Step 3: Remove the protractor and with a ruler draw a line passing through points A and B.

Then, AB ⊥ line *l* at the given point A. How many such lines can you draw at A which are perpendicular to line *l*? Only one.

(b) Using a set-square and a ruler: For this construction, we go through the following steps:

Step 1: Place the ruler on the paper with one of its long edges lying on the given line *l*.

Step 2: Holding the ruler fixed, place a set-square PQR with one arm PR of its right angle P in contact with the ruler [Fig. 12.56(i)].

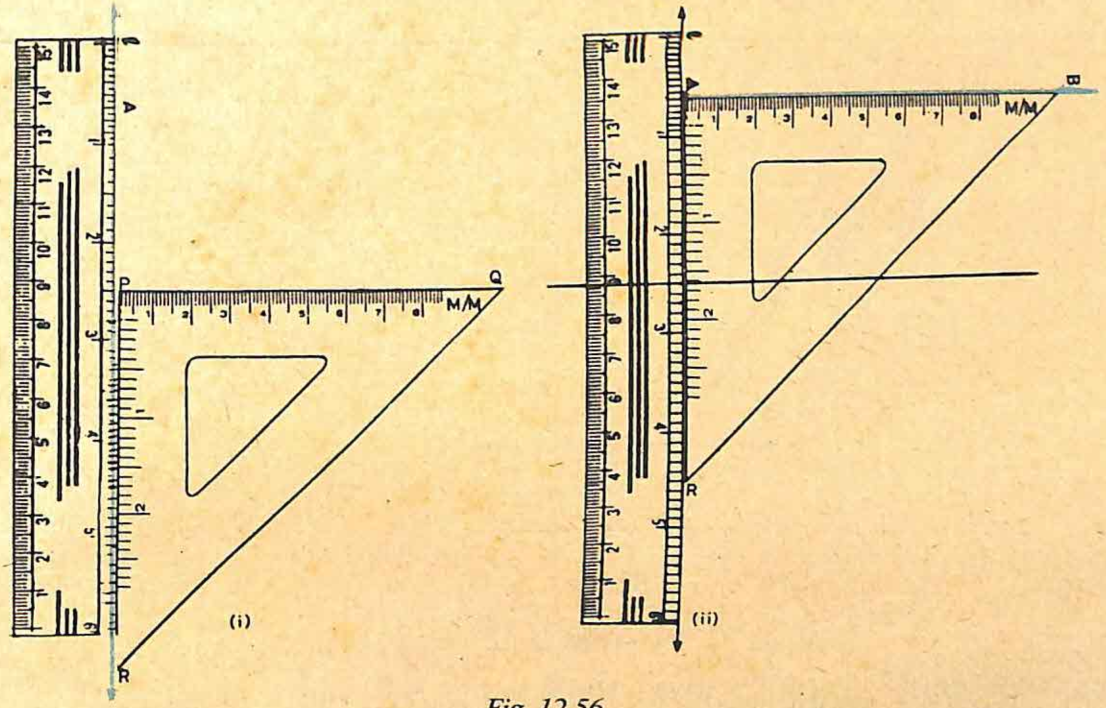


Fig. 12.56

Step 3: Slide the set-square along the edge of the ruler until P coincides with given point A [Fig. 12.56(ii)].

Step 4: Holding the set-square fixed in this position, we draw with a sharp pencil a line AB along the edge PQ.
Then, line AB ⊥ line *l* at A.

(ii) Construction of a line perpendicular to a given line l and passing through a given point A not lying on it, using a set-square

For this construction we go through the following steps:

Step 1: Place either of the set-squares so that one arm (edge) PR of the right angle is along *l* [Fig. 12.57(i)].

Step 2: Hold the set-square fixed and place a ruler along the edge QR opposite to the right angle of the set-square.

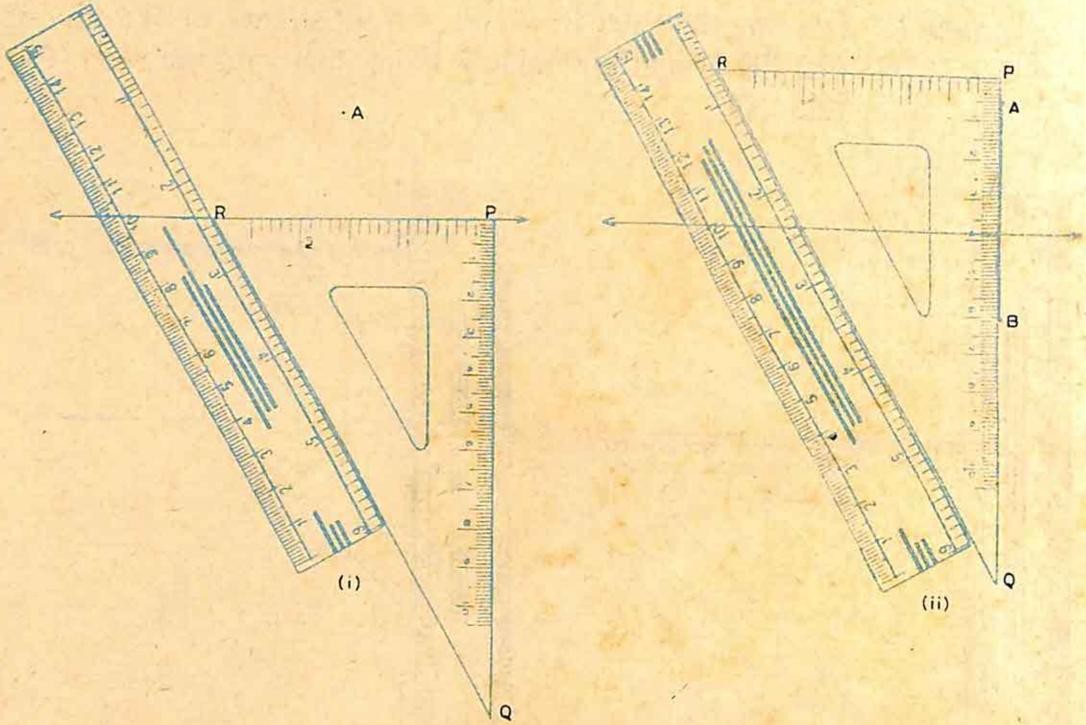


Fig. 12.57

Step 3: Holding the ruler fixed, slide the set-square along the ruler until the other arm PQ of the right angle passes through the given point A [Fig. 12.57(ii)].

Step 4: Keeping the set-square fixed in this position, draw a line AB along edge PQ.

Then, AB is the line perpendicular to *l* and passing through A. How many such lines can you draw? Only one.

EXERCISE 12.6

1. How many lines can be drawn which are perpendicular to a given line and pass through a given point lying
   (i) outside it? (ii) on it?

2. Draw a line AB. Take a point C not lying on it. Using set-squares construct a line ED ⊥ AB which passes through point C. Check with a protractor whether the angle between the lines is 90°.
3. Check by a protractor or a set-square which of the following are perpendicular to each other?

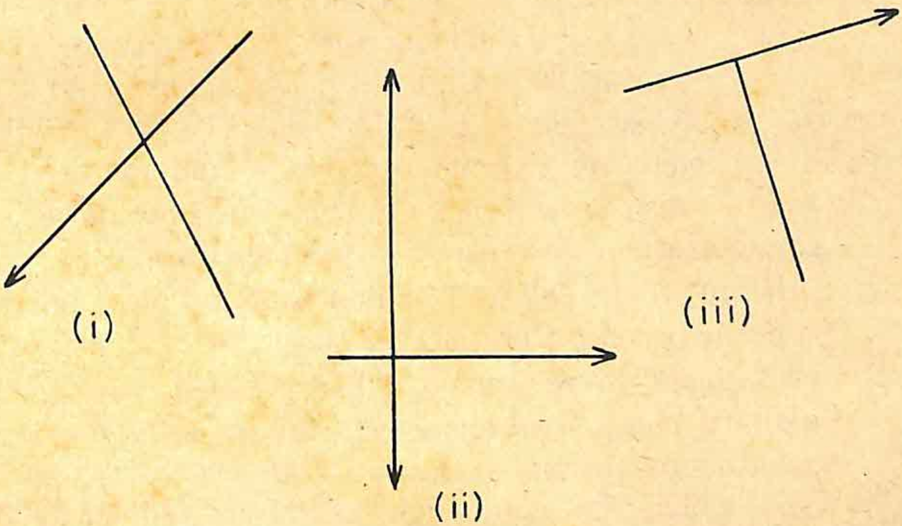


Fig. 12.58

4. Draw a line AB and take a point C on it. Using set-squares, construct a perpendicular CD on it. Check by using a protractor whether ∠ACD = 90°.
5. Check by using a set-square whether the angle at a corner of a sheet of paper of your notebook is a right angle.

Things to Remember

1. Ray AB has one initial point (or end-point) A and extends indefinitely in one direction. Ray AB and ray BA are two different rays.
2. Two rays with same initial point but directed in the opposite directions of the same line are called opposite rays.

3. An angle is a figure formed by two rays with the same initial point. Initial point is the vertex and two rays are the arms of the angle.
4. Two or more angles having the same magnitude are said to be equal.
5. 1 complete angle = 4 right angles = $360°$, 1 straight angle = 2 right angles = $180°$, 1 right angle = $90°$, zero angle = $0°$, $0° <$ acute angle $< 90°$, $90° <$ obtuse angle $< 180°$, $1° = 60'$, $1' = 60''$.
6. Two angles with a common vertex, a common arm and the other arms lying on the opposite sides of the common arm form a pair of adjacent angles.
7. Adjacent angles whose two non-common arms are opposite rays form a linear pair.
8. Two angles whose sum is $90°$ are called complementary angles. Two angles whose sum is $180°$ are called supplementary angles.
9. Lines which intersect at right angles are said to be perpendicular to each other.
10. Rays and segments are said to be perpendicular if the corresponding lines determined by them are perpendicular.
11. Only one line can be drawn perpendicular to a given line which passes through a given point not lying on it.
12. In a plane only one line can be drawn perpendicular to a given line at a given point on it.
13. If two lines intersect, then vertically opposite angles are equal.
14. Two angles forming a linear pair are adjacent and supplementary.
15. Two supplementary angles may not form a linear pair.

CHAPTER THIRTEEN

Parallel Lines

IN THIS CHAPTER, we shall study about parallel lines. We shall also learn about distance between them, transversals, angles made by a transversal with two lines, relations between angles made by a transversal with two parallel lines and construction of parallel lines.

13.1 Parallel Lines

Recall that two lines which are in the same plane and do not intersect are called parallel lines. In Fig. 13.1, lines l and m are parallel lines. We write $l \parallel m$ and read 'l is parallel to m'.

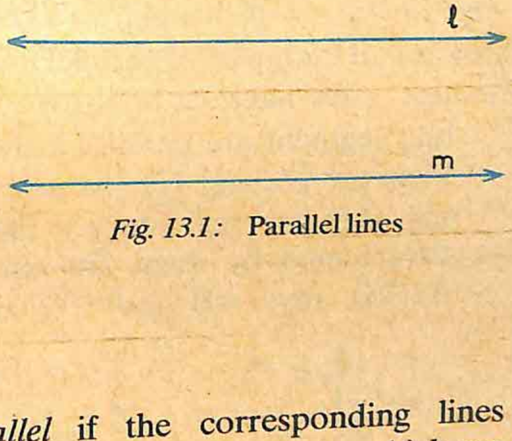
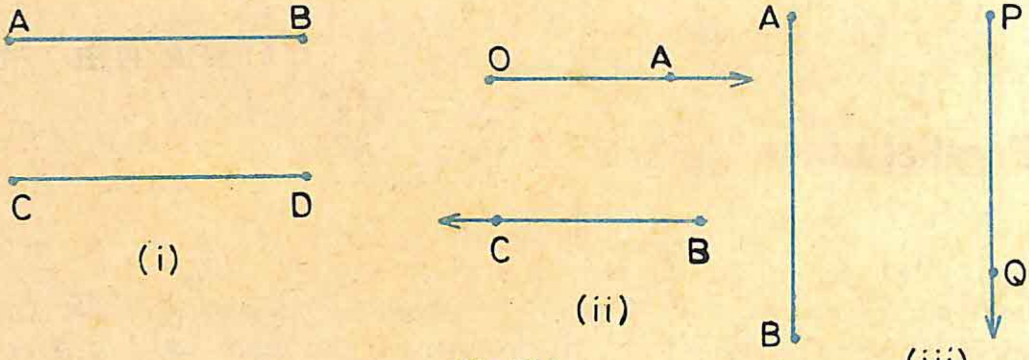


Fig. 13.1: Parallel lines

Recall that in Section 12.11 we have said about the perpendicularity of two rays, two line segments, one ray and one line segment. In the same way, here we say:

(i) *Two segments are parallel* if the corresponding lines determined by them are parallel, i.e. two segments which are in the same plane and do not intersect each other even if extended indefinitely in both directions are said to be parallel. For example, in Fig. 13.2(i), segment AB ∥ segment CD.

(ii) *Two rays are parallel* if the corresponding lines determined by them are parallel, i.e. two rays which are in the same plane



(i) (ii) (iii)

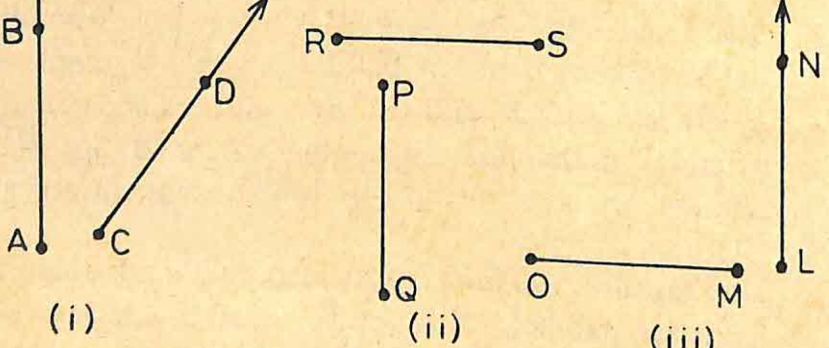
Fig. 13.2

and do not intersect each other even if extended indefinitely beyond their initial points are said to be parallel. In Fig. 13.2 (ii), ray OA ∥ ray BC.

(iii) *One segment and one ray* are *parallel* if the corresponding lines determined by them are parallel. In Fig. 13.2(iii), segment AB ∥ ray PQ.

Opposite edges of this paper, a ruler, a wall, a ceiling, a black-board, a floor are all examples of parallel line segments.

Remark: We have said that two rays or two segments or one ray and one segment are parallel if the corresponding lines determined by them are parallel. Can we say that two rays or two segments or one ray and one segment are intersecting if the corresponding lines determined by them are intersecting? No. For example, in Fig. 13.3(i), rays AB and CD are non-intersecting, in Fig. 13.3(ii),



(i) (ii) (iii)

Fig. 13.3

segments PQ and RS are non-intersecting and in Fig. 13.3(iii), segment OM and ray LN are non-intersecting.

Thus, two segments in a plane are either intersecting or non-intersecting. But if they do not intersect, then we cannot say that they are parallel. The same is true for two rays, one ray and one segment, etc.

13.2 Distance between Parallel Lines

Let us perform the following experiments:

Experiment 1: Let us consider two lines l and m which are not parallel. We mark two points A and B on l and m, respectively, very close to the point of intersection O. Join them by a line segment AB (Fig. 13.4). Now mark two more points C and D as shown in the figure on l and m. Join C and D. Similarly, we take other points on l and m and join them.

Compare lengths of segments AB, CD, EF, QP, ... What do you observe? We see that $AB < CD < EF$ and $QP < RS < UT$. We observe that separation between the lines increases as we move on the lines away from the point of intersection O.

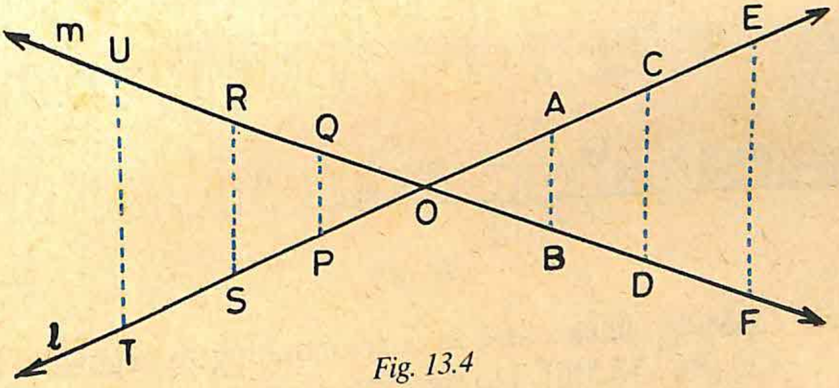
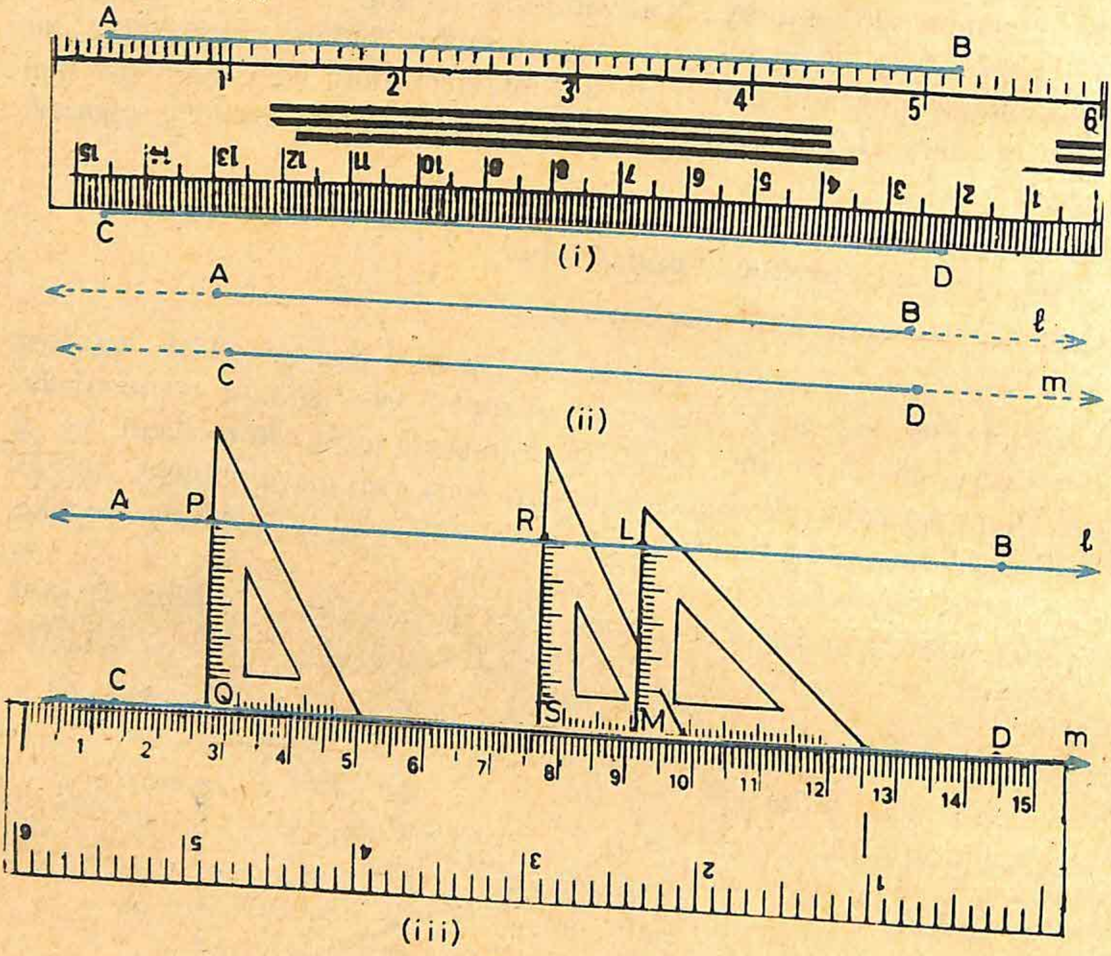


Fig. 13.4

Can we speak of the distance between two intersecting lines l and m?

No, because the separation between the lines is not constant.

Experiment 2: (i) Let us place a straight-edged ruler on a plane sheet of paper and hold it firmly with one hand. We draw with a sharp pencil two line segments AB and CD along the longer edges of the ruler [Fig. 13.5(i)].



(i)

(ii)

(iii)

Fig. 13.5

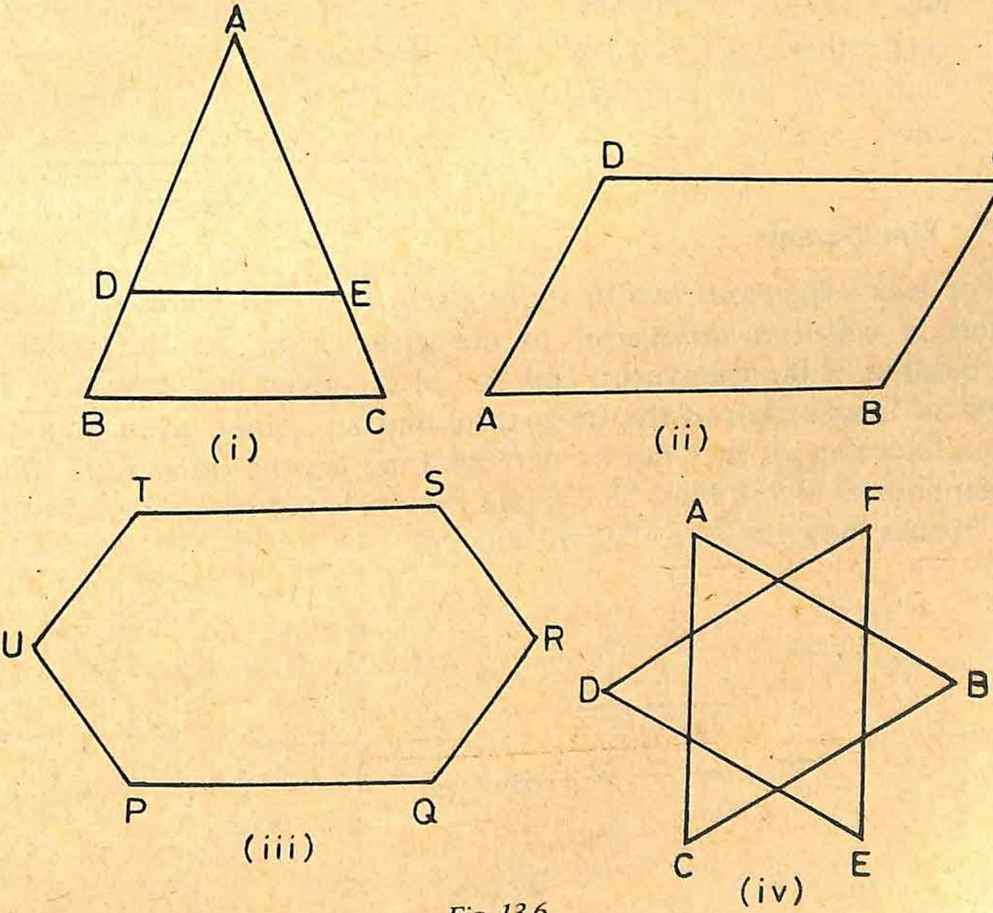
(ii) Consider lines *l* and m corresponding to segments AB and CD [Fig. 13.5(ii)]. Do they intersect? No. (We can check it by extending segments AB and CD on both sides). Are they in the same plane? Yes. Hence *l* || m.

(iii) Place the ruler so that one of its edges is along CD. Place a set-square with one arm of the right angle coinciding with the edge of the ruler. Draw line segment PQ along the edge of the set-square as shown in Fig. 13.5(iii). Slide the set-square along the ruler and draw some more segments RS, LM joining the two parallel lines l and m. Measure lengths of PQ, RS and LM. Are they equal? Yes. IS PQ $\perp$ l? Yes. Is PQ $\perp$ m? Yes. Thus, we see that PQ is the perpendicular distance between l and m. The same is true for RS, LM, etc.

From this experiment, we observe that the *perpendicular distance between two parallel lines is the same everywhere.* We call this distance as *the distance between two parallel lines.*

EXERCISE 13.1

1. Identify parallel line segments in Fig. 13.6.



(i) (ii) (iii) (iv)

Fig. 13.6

2. Some parallel lines are shown in Fig. 13.7.
   (i) Find the distance between lines *l* and m using a set-square.

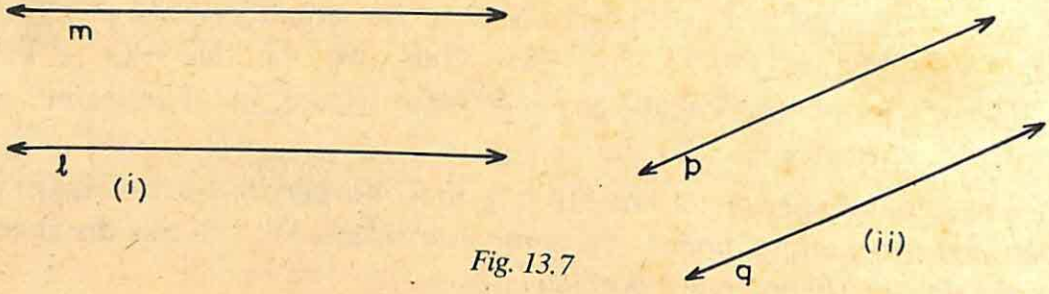


Fig. 13.7

   (ii) Find the distance between p and q using a set-square.

3. In Fig. 13.8, segments AB and CD do not intersect each other. Can you say that they are parallel? Give reasons.

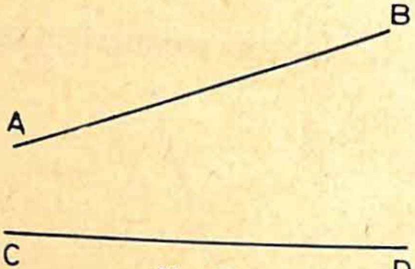


Fig. 13.8

13.3 Transversals

A line which intersects two or more given lines in a plane at different points is called a transversal to the given lines. So the point of intersection of the transversal and one of the given line cannot be the point of intersection of the transversal and any other given line. The given lines may or may not be parallel. Line *l* is the transversal to the given lines shown in Figs. 13.9(i)-(vi). Line m is not a transversal to the given lines shown in Figs. 13.9(vii) and (viii).

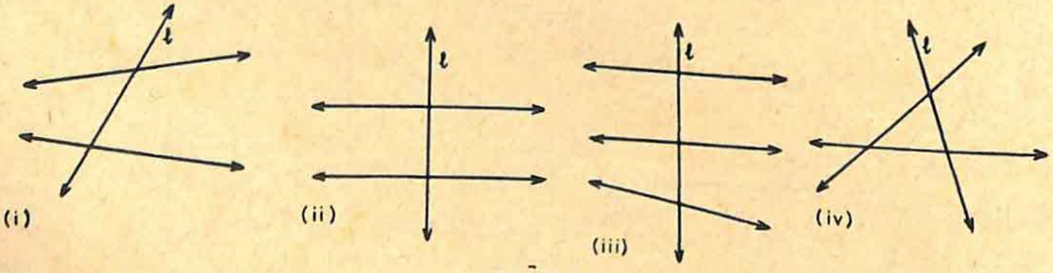


Fig. 13.9

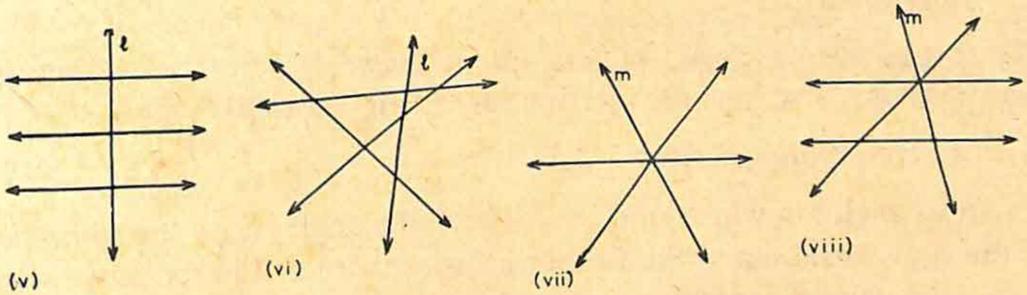


Fig. 13.9

13.4 Angles Made by a Transversal with Two Lines

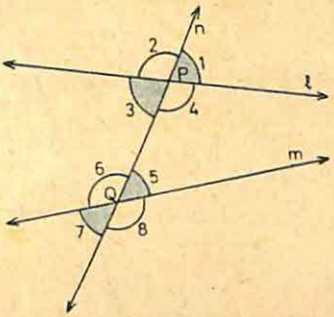
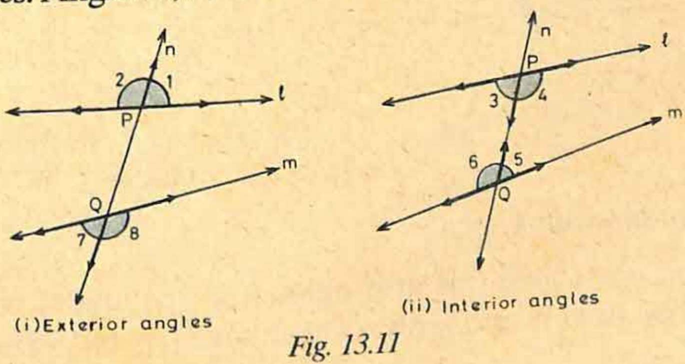


Fig. 13.10

Let us draw two lines *l* and m and a transversal n intersecting *l* and m at P and Q, respectively (Fig. 13.10). How many angles are formed by the transversal n with each line? Four. So a total of eight angles are formed by a transversal with two given lines. Label them 1 to 8, for convenience. These angles are given special names as follows:

(i) Exterior Angles

The angles whose arms do not contain line segment PQ are called exterior angles. Angles 1, 2, 7, 8 are exterior angles [Fig. 13.11(i)].



(i) Exterior angles

(ii) Interior angles

Fig. 13.11

(ii) Interior Angles

The angles whose arms include line segment PQ are called interior angles. Angles 3, 4, ,5, 6 are interior angles [Fig. 13.11(ii)].

(iii) Corresponding Angles

A pair of angles in which one arm of both the angles is on the *same side* of the transversal n and their other arms are directed in the *same sense* is called a pair of *corresponding angles.* For example, ∠ 1 and ∠ 5 form a pair of corresponding angles (Fig. 13.12). There are four possible pairs of corresponding angles [Fig. 13.12(i)-(iv)].

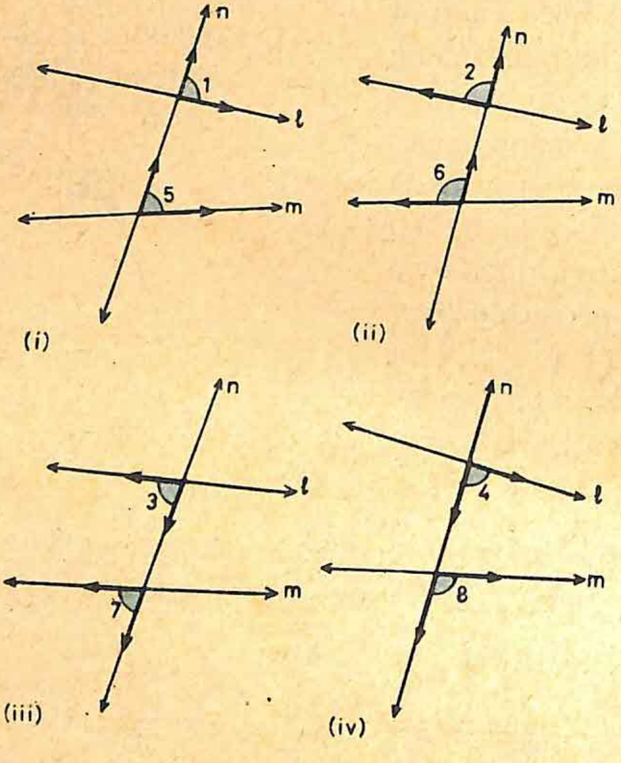


Fig. 13.12

(iv) Alternate Interior Angles

A pair of angles in which one arm of each of the angles is on opposite sides of the transversal and whose other arms include segment PQ *as* *shown in the figure* form a pair of *alternate interior angles.* For example, ∠3 and ∠5 form a pair of alternate interior angles [Fig 13.13(i)]. Angles 4 and 6 form another pair of alternate interior angles [Fig. 13.13(ii)].

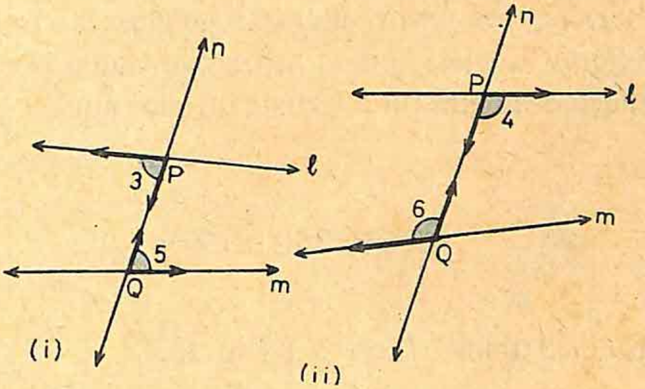


Fig. 13.13

(v) Alternate Exterior Angles

A pair of angles in which one arm of each of the angles is on opposite sides of the transversal and whose other arms are directed in *opposite sense* and *do not* include segment PQ is called a pair of *alternate exterior angles.* For example, in Fig. 13.14, ∠2 and ∠8, ∠1 and ∠7 form pairs of alternate exterior angles.

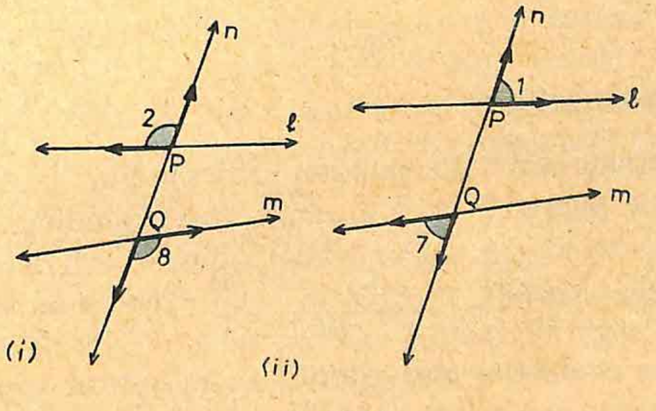


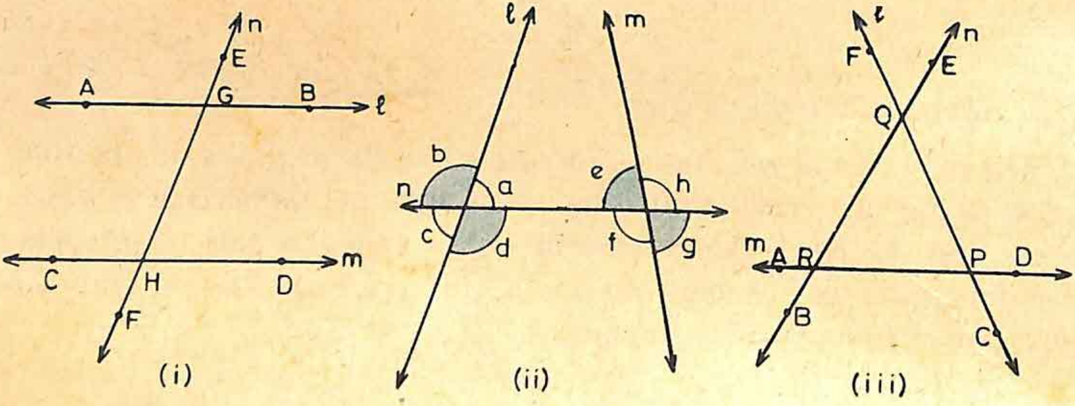
Fig. 13.14

Further, each pair of angles 3, 6 and 4, 5 are called a pair of *interior angles* on the *same side* of transversal. Similarly, angles 1, 8 and 2, 7 are pairs of exterior angles on the same side of transversal as shown in Fig. 13.10.

Remark: Generally, the term alternate angles is used to indicate alternate interior angles. Unless otherwise stated, we shall also use the term alternate angles for alternate interior angles.

EXERCISE 13.2

Line n is a transversal to lines *l* and m in Fig. 13.15.



(i) (ii) (iii)

Fig. 13.15

Identify the following angles:

(i) Alternate and corresponding angles in Fig. 13.15(i).

(ii) Angle alternate to ∠d, angle corresponding to ∠f, angle alternate to ∠g, angle corresponding to ∠h in Fig. 13.15(ii).

(iii) Angle alternate to ∠PQR, angle corresponding to ∠RQF and angle alternate to ∠PQE in Fig. 13.15(iii).

(iv) Pairs of interior and exterior angles on the same side of the transversal n in Fig. 13.15(ii).

13.5 Angles Made by a Transversal to Two Parallel Lines

Let us perform the following experiments.

Experiment 1: (a) Using a ruler, let us draw two parallel lines *l* and m (Recall Experiment 2, Section 13.2).

(b) We draw a transversal n such that it intersects lines *l* and m at the points E and F, respectively (Fig. 13.16).

(c) Measure corresponding angles 1 and 5 with the help of a protractor. Are they equal? Again measure corresponding angles 2, 6; 3, 7 and 4, 8. What do you observe? Are the corresponding angles equal? Yes. ∠1 = ∠5, ∠2 = ∠6, ∠3 = ∠7 and ∠4 = ∠8.

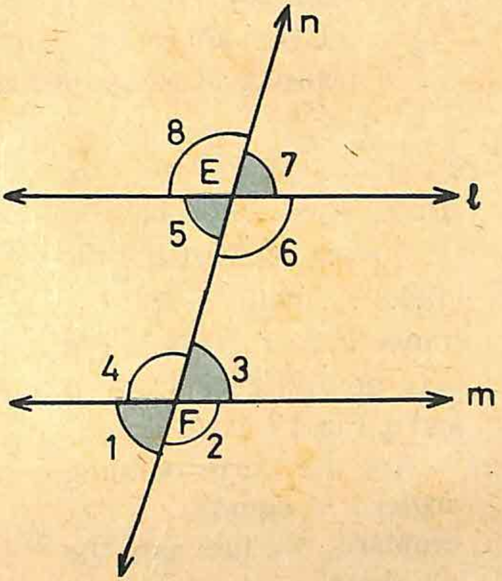


Fig. 13.16

(d) Now measure alternate interior and exterior angles 3, 5; 4, 6; 1, 7; 2, 8. Are these pairs of angles equal? Yes.

(e) Find the measures of the interior angles 3 and 6 lying on the same side of the transversal and add them. Do you get the sum equal to 180°? Yes.

(f) Similarly, find the sum of the angles 4 and 5 and verify that it is also equal to 180°. Also find the sum of the exterior angles 2, 7 and 1, 8. In each case, verify that the sum is 180°.

We can repeat this experiment by drawing another pair of parallel lines and a transversal to it. We will obtain the same results.

From the above experiment, we observe the following properties of angles formed by a transversal to two parallel lines:

(i) Pairs of alternate (interior or exterior) angles are equal.

(ii) Pairs of corresponding angles are equal.

(iii) The sum of the interior angles (or exterior angles) on the same side of the transversal is 180°. In other words, the interior angles (or exterior angles) on the same side of the transversal are supplementary.

Experiment 2: Let us draw two non-parallel lines p and q. Measure the angles made by a transversal r with these two non-parallel lines p and q [Fig. 13.17].

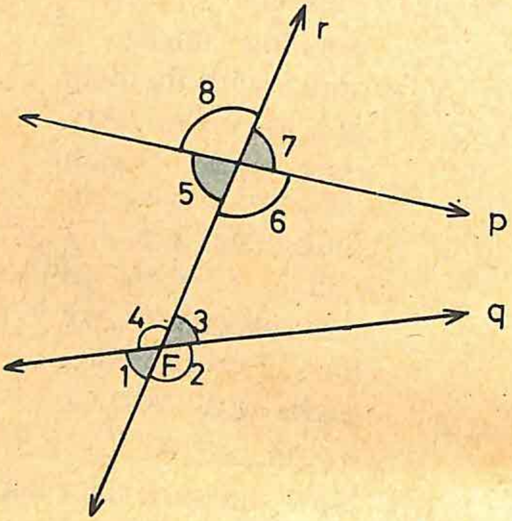


Fig. 13.17

Are the corresponding angles equal? No. Similarly, we find that the alternate angles are not equal. Is the sum of interior angles (or exterior angles) on the same side of the transversal 180°? No. Thus, we observe that none of the above properties (i) to (iii) which are true for parallel lines hold for non-parallel lines. Moreover, *if two lines are intersected by a transversal and any of the properties (i) to (iii) is true, then the lines will be parallel.* We illustrate these properties through an example.

Example: In Fig. 13.18, line $l \parallel$ line m, n is a transversal and $\angle 1 = 40^\circ$. Find all the other angles marked in the figure.

Solution: Since, $\angle 1$ and $\angle 5$ are corresponding angles, therefore, $\angle 5 = \angle 1 = 40°$.

$\angle 1$ and $\angle 2$ form a linear pair.

Therefore, $\angle 2 = 180° - \angle 1 = 180° - 40° = 140°$

Again, $\angle 5$ and $\angle 3$ are alternate angles.

Therefore, $\angle 3 = \angle 5 = 40°$

Similarly,

$\angle 4 = 180° - \angle 3$ (linear pair)

$= 180° - 40°$

$= 140°$

$\angle 6 = \angle 4$

$= 140°$ [alternate angles]

$\angle 7 = \angle 3 = 40°$ [corresponding angles].

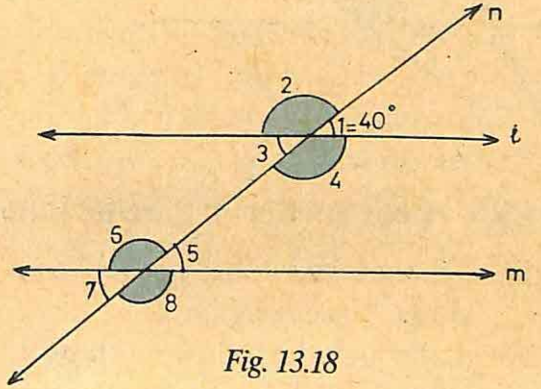


Fig. 13.18

EXERCISE 13.3

1. In Fig. 13.19, AB and CD are parallel lines intersected by a transversal PQ at L and M, respectively. If $\angle CMQ = 60°$, find all other angles in the figure.

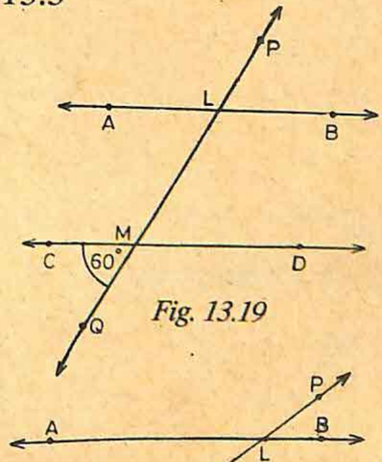


Fig. 13.19

2. In Fig. 13.20, AB and CD are parallel lines intersected by a transversal PQ at L and M, respectively. If $\angle LMD = 35°$, find $\angle ALM$ and $\angle PLA$.

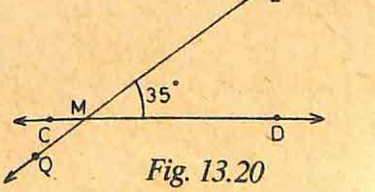


Fig. 13.20

3. Find x in Fig. 13.21 for each figure if $l \parallel m$ and $p \parallel q$.

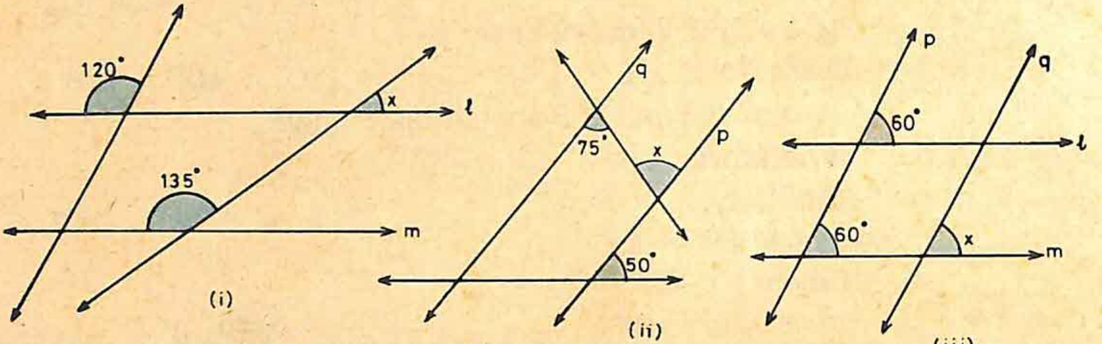


Fig. 13.21

13.6 Construction of Parallel Lines Using a Set-square and a Ruler

(i) To construct a line parallel to a given line l passing through a given point A not lying on it.

We go through the following steps to do this construction:

Step 1: We place a set-square with one arm of its right angle along *l* as shown in Fig. 13.22 (i).

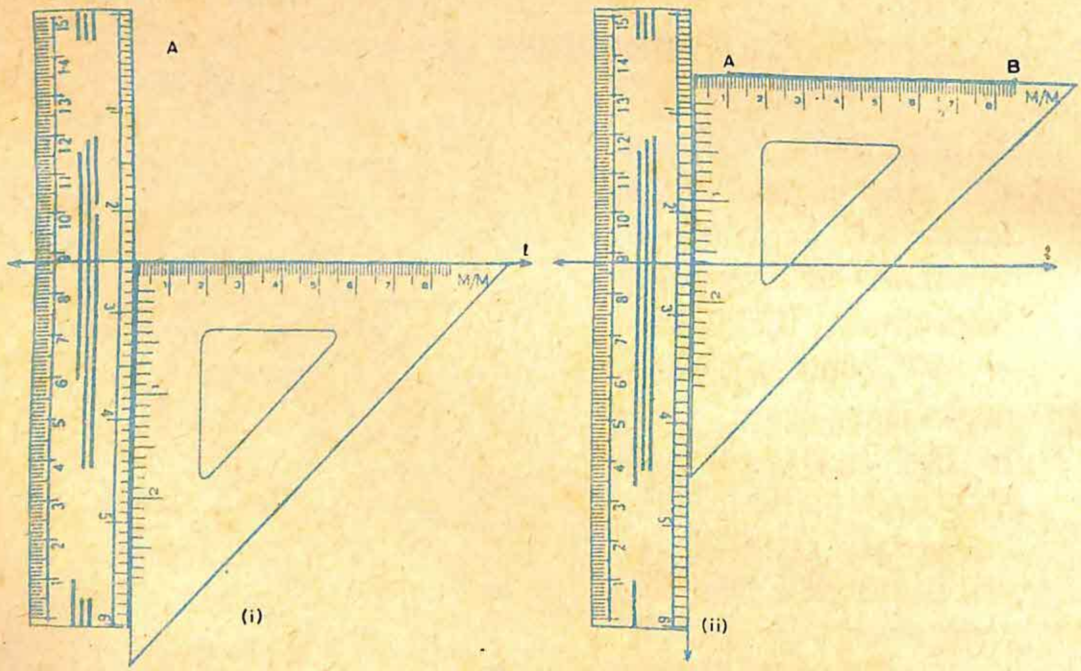


Fig. 13.22

Step 2: Holding the set-square fixed, we place a ruler along the other arm of the right angle.

Step 3: Holding the ruler fixed, we slide the set-square along the edge of the ruler until the *first arm* (edge) of the right angle of the set-square passes through the given point A [Fig. 13.22 (ii)].

Step 4: Keeping the set-square fixed in this position, we draw a line AB through the point A along this edge.

Then, AB is the required line through A and parallel to *l*.

Remark: Why is line AB ∥ line *l*? Because interior angles on the same side of the transversal are supplementary (90° + 90°). How many lines can you draw through A which are parallel to *l*? Only one.

(ii) *To construct a line parallel to a given line l at a given distance d, say 4 cm, from it*

We go through the following steps to do this construction:

Step 1: We mark a point A on *l* [Fig. 13.23].

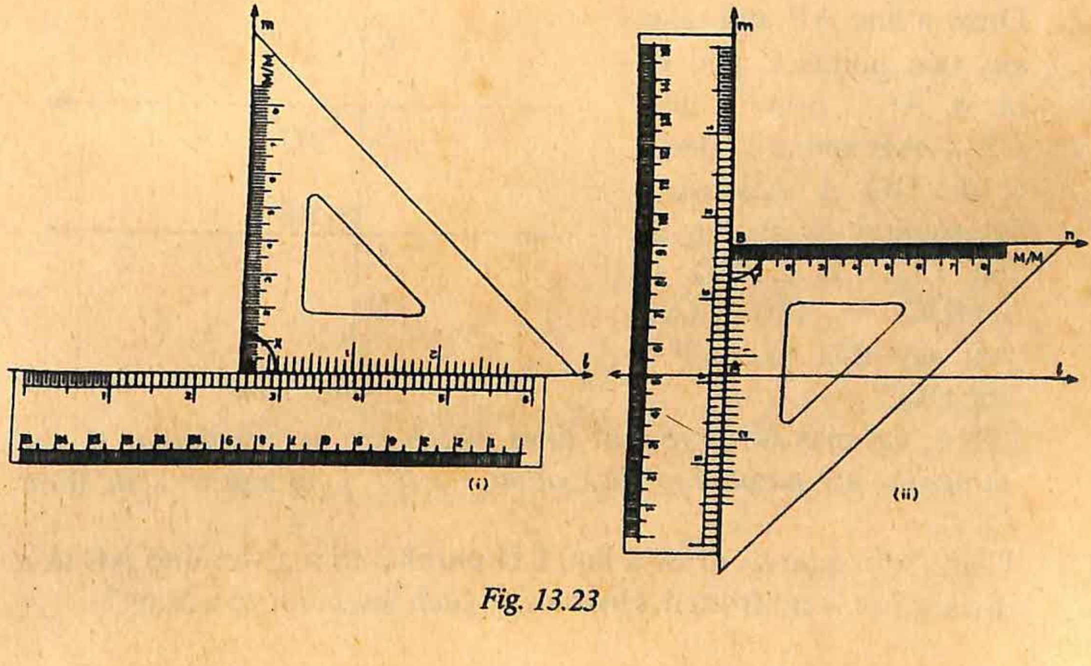


Fig. 13.23

Step 2: Using set-squares, we construct a line m perpendicular to l at A.

Step 3: With a ruler or compasses mark a point B on m such that segment AB = 4 cm.

Step 4: Using set-squares construct a line n perpendicular to m at B.

Then, n is the required line parallel to l and is at a distance of 4 cm from it. The lines n and l are parallel because interior angles x and y on the same side of the transversal m are supplementary.

EXERCISE 13.4

1. Draw a line AB and take two points C and E on its opposite sides. Through C, draw CD ∥ AB and through E draw EF ∥ AB, using set-squares. Use set-squares to check that CD ∥ EF.
   [Thus, we may observe that *lines which are parallel to the same line are parallel to each other,* i.e. if $l \parallel m$ and $n \parallel m$, then $l \parallel n$.]
2. Draw a line AB and take any two points C and D on it. At C draw a line CP ⊥ AB and at D draw a line DQ ⊥ AB using set-squares as shown in Fig. 13.24. Is ∠PCD + ∠QDC = 180°? Can you say that line CP ∥ line DQ?

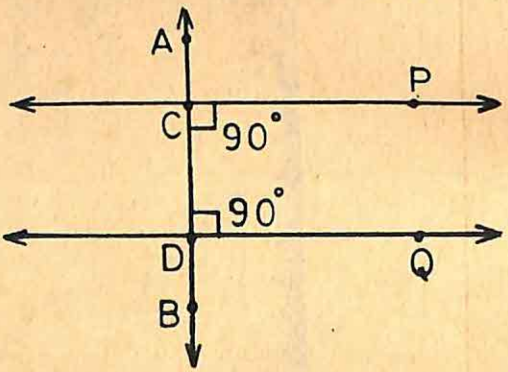


Fig. 13.24

   [Thus, we may observe that *lines which are perpendicular to the same line are parallel to each other, i.e. if* $l \perp m$ and $n \perp m$, then $l \parallel n$.]
3. Using set-squares, draw a line CD parallel to a given line AB at a distance of 3 cm from it. How many such lines can you draw?

Things to Remember

1. Lines which do not intersect in a plane are called parallel lines.
2. The distance between two parallel lines is the same everywhere.
3. A line which intersects two or more given lines at different points is called a transveral to the given lines.
4. If two parallel lines are intersected by a transversal, then
    (a) all pairs of corresponding angles are equal.
    (b) all pairs of alternate angles are equal.
    (c) interior angles (or exterior angles) on the same side of the transversal are supplementary.
5. If two non-parallel lines are intersected by a transversal, then (a), (b), (c) of 4 are not true.
6. If two lines are intersected by a transversal and if *any one* of the following is true:
    (a) One pair of corresponding angles are equal,
    (b) One pair of alternate angles are equal,
    (c) A pair of interior angles (or exterior angles) on one side of the transversal are supplementary,

    then the two lines are parallel.
7. Lines parallel to the same line are parallel to each other.
8. Lines perpendicular to the same line are parallel to each other.

UNIT FIVE

STATISTICS

INITIALLY, THE WORD 'STATISTICS' used to mean simply the collection of information in the form of numerical data by the kings (or governments) on different aspects such as population, property, wealth, etc. of their respective states. These data were useful to the kings in assessing the real situations in their respective states and consequently taking possible steps such as imposition of taxes and levies, proper utilization of the manpower and natural resources for the welfare of the people of the state. The existence of this practice of collecting numerical data in ancient India is evident from the fact that during the reign of Chandragupta Maurya (324 B.C.-300 B.C.), there was a very good system of collecting such data specially in regard to the births and deaths. Thus, originally statistics was confined only to the affairs of the state. However, with the passage of time, its scope was widened and it began to include collection of numercial data from almost every sphere of life (such as exports and imports, marriages and divorces, the daily maximum and minimum temperatures, cost of living, etc.), calculation of percentages, etc. and presentation of data in tabular and pictorial forms. By the end of the nineteenth century, the scope of statistics was further widened and it began to concern itself not only with the collection and presentation of data but also with interpretation and drawing inferences from the data so obtained.

In the modern age, it is difficult to imagine any activity of our life untouched by numerical data. It is, therefore, essential to know differtent methods to extract useful information from these data. For this, data are presented in the form of tables and pictures (or graphs). Data are usually presented pictorially by pictographs, bar graphs, circle graphs (pie charts), etc. In Chapter Fourteen, we are making a beginning to the study of statistics with the reading and interpretation of bar graphs.

CHAPTER FOURTEEN

Bar Graphs

IN THIS CHAPTER, bar graphs have been introduced after giving a brief explanation of numerical data and pictographs. We shall learn to read and interpret some given bar graphs based on the data selected from various aspects of life.

14.1 Introduction

Let us look carefully at the following information:

Traffic at a busy road crossing in Delhi was studied by the Traffic Police on a particular day. The approximate number of vehicles passing through the crossing every hour for nine time intervals has been recorded in the following table:

TABLE 14.1

Number of Vehicles Passing through a Busy Road Crossing in Delhi in Different Time Intervals on a Particular Day

Time interval	8 to 9 Hrs	9 to 10 Hrs	10 to 11 Hrs	11 to 12 Hrs	12 to 13 Hrs	13 to 14 Hrs	14 to 15 Hrs	15 to 16 Hrs	16 to 17 Hrs
Number of vehicles	300	400	250	250	150	100	200	300	350

Such type of information given with the help of numbers is called a *numerical data.* To study the details and characteristics of numerical data pictures are quite helpful. The study of numerical data through pictures or graphs is known as the *pictorial representation or graph of the data.*

There are many methods of representing numerical data pictorially. One of the methods is to use picture symbols. Such a representation is called a *pictograph.*

Let us represent the information given in Table 14.1 through a pictograph (Fig. 14.1).

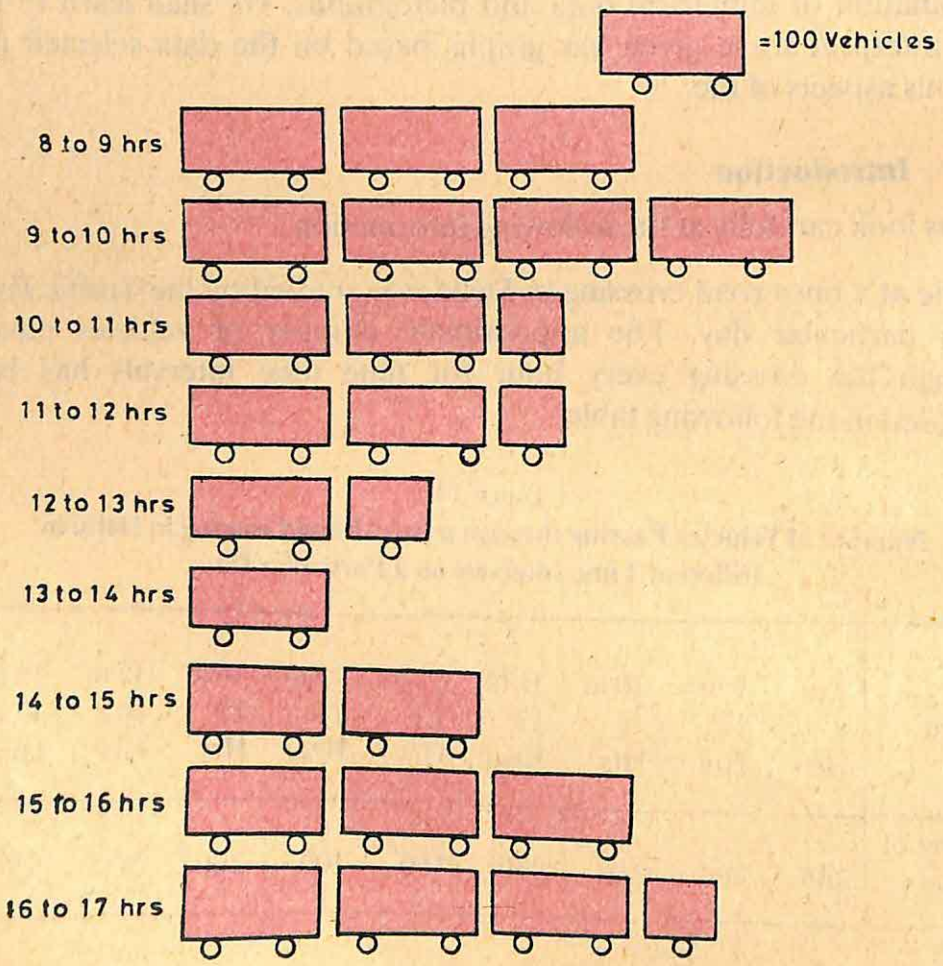


Fig. 14.1: Pictograph of the number of vehicles passing through a busy road crossing in Delhi in different time intervals on a particular day

This pictograph reveals to an eye a pattern in the vehicular traffic at the road crossing. The picture symbol represent 100 vehicles. But this way of representing numerical data is time-consuming and, therefore, not very useful. Further, many a time we will have to draw part picture symbols which is also a difficult task. Thus, instead of picture symbols, it is easy to draw bars (rectangles) representing the given number of vehicles as shown in Fig. 14.2.

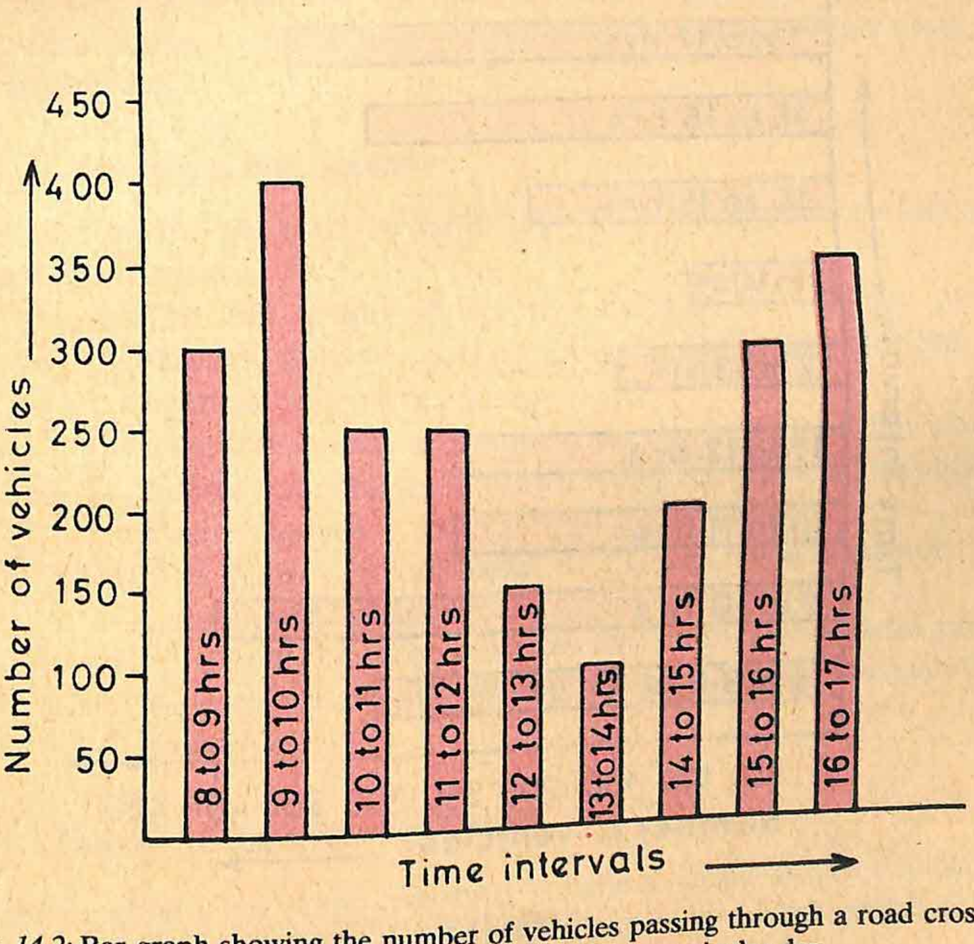


Fig. 14.2: Bar graph showing the number of vehicles passing through a road crossing in Delhi in different time intervals on a particular day

Such a representation of data with the help of bars in a diagram is called a *bar graph* or a *bar chart*. The time intervals have been arranged horizontally and the bars are vertical. The bars can be shaded, hatched or coloured. Note that the graph can also be presented by placing the time intervals vertically and bars horizontally (Fig. 14.3). Generally, vertical bar graphs are preferred.

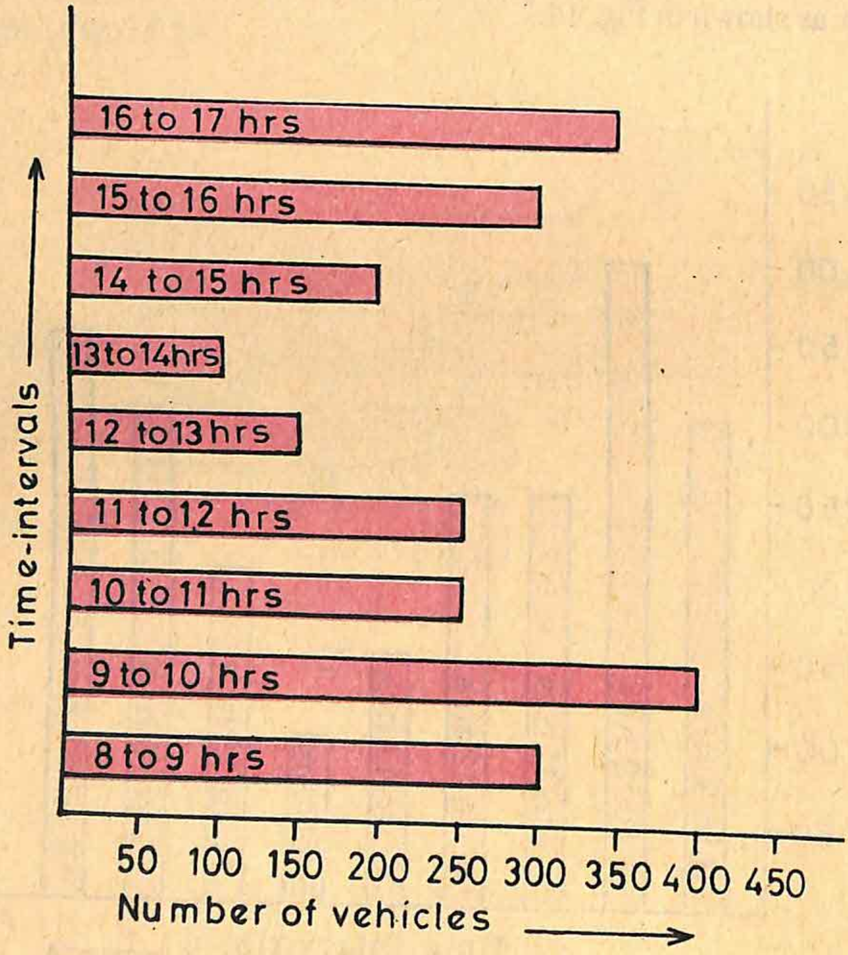


Fig. 14.3

You must have seen similar bar graphs in newspapers, magazines, government reports, advertisements, etc. These are useful in bringing out salient features of numerical data.

A bar graph is a pictorial representation of the numerical data by a number of bars (rectangles) of uniform width erected horizontally or vertically with equal spacing between them. Each rectangle or bar represents only one value of the numerical data and so there are as many bars as the number of values in the numerical data. The height or length of a bar indicates on a suitable scale the corresponding value of the numerical data. If the bars are drawn on the horizontal line (we may call it x-axis), then the scale of heights of the bars or rectangles is shown along the vertical line (we may call it y-axis). If the bars are drawn on the vertical line, then the scale of heights of the bars is shown along the horizontal line.

14.2 Reading of Bar Graphs

Let us read the bar graph given in Fig. 14.2. We observe the following from this bar graph:

(i) The bar graph shows the number of vehicles passing through a busy road crossing in Delhi in different time intervals on a particular day.

(ii) The time break-up is shown on the horizontal line and the number of vehicles is shown along the vertical line choosing a suitable scale.

(iii) The scale is 1 cm equal to 50 units which means 1 cm indicates 50 vehicles.

(iv) The height of a bar indicates the number of vehicles passing through the crossing during the time interval corresponding to the bar.

14.3 Interpretation of the Bar Graph

After reading a bar graph one must be able to draw certain conclusions from it. That is what we mean by interpretation of bar graph. Let us draw certain conclusions from bar graph of Fig. 14.2.

(i) What does the bar graph represent? The bar graph represents the number of vehicles passing through a particular crossing of Delhi in different time intervals on a particular day.

(ii) When is the hourly traffic maximum? The maximum traffic corresponds to the longest bar and occurs between 9 to 10 hours. The number of vehicles is 400.

(iii) Similarly, the minimum traffic corresponds to the shortest bar and it occurs between 13 to 14 hours. The number of vehicles is 100.

(iv) The total traffic during two peak hours of morning rush for schools, offices and business establishments as shown by long bars in the morning time is 300 + 400 = 700 vehicles. Simiarly, the large bars in the evening at 15 to 16 and 16 to 17 hours show the peak hours of evening traffic. Thus, we observe that bar graphs help in easy comprehension of a given data. Conclusions can be drawn at a glance. The graphs are more attractive and appealing than the data.

Now let us consider a few more examples of reading and interpreting bar graphs.

Example 1: Read the bar graph represented in Fig. 14.4 and answer the following questions:

(i) What is the information given by the bar graph?

(ii) What is the order of the change of number of students over several years?

(iii) In which year is the increase of number of students maximum?

(iv) State whether true or false:
The enrolment during 1982-83 is twice that of 1981-1982.

Solution: The answers to the above questions are:

(i) The bar graph represents the number of students in Class VI of a school during academic years 1981-82 to 1985-86.

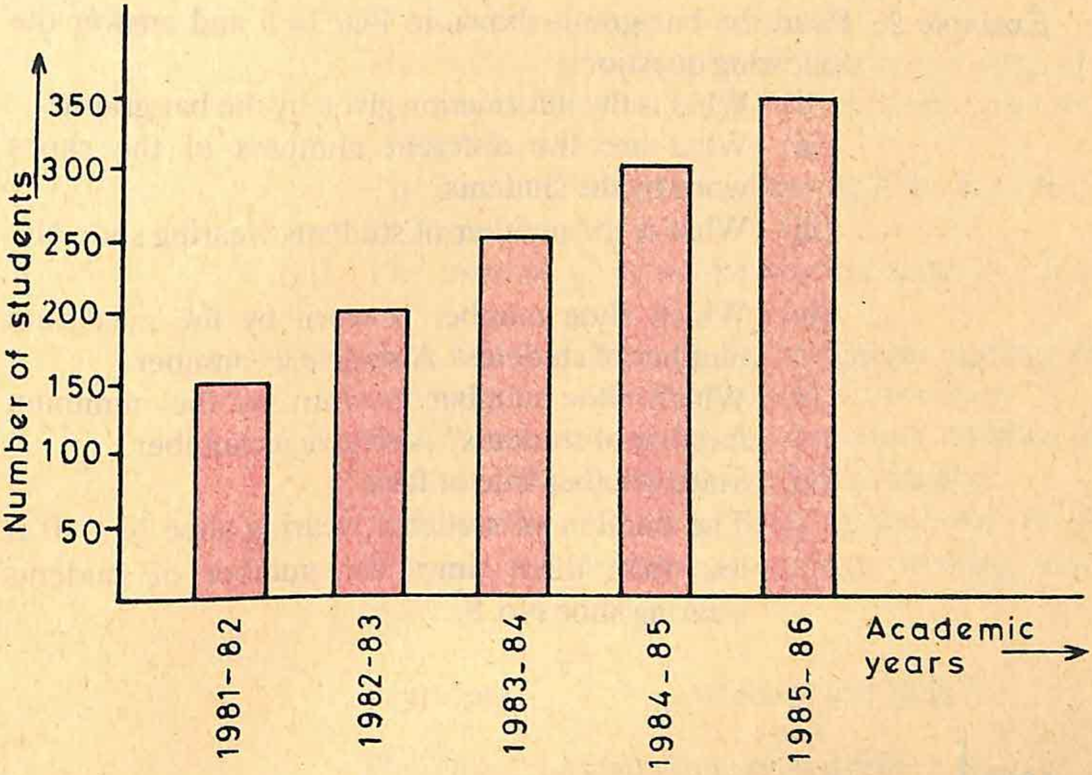


Fig. 14.4: Bar graph of the number of students in Class VI of a school during academic years 1981-82 to 1985-86

(ii) The number of students is changing in an increasing order as the heights of bars are increasing.

(iii) The increase in the number of students is uniform as the increase in the height of bars is uniform. Hence, in this case, the increase is not maximum in any of the years.

(iv) Enrolment in 1982-83 = 200 and enrolment in 1981-82 = 150

$$\frac{200}{150} = \frac{4}{3} = 1\frac{1}{3} < 2$$

Therefore, the statement is false.

Example 2: Read the bar graph shown in Fig. 14.5 and answer the following questions:

(i) What is the information given by the bar graph?

(ii) What are the different numbers of the shoes worn by the students?

(iii) What is the number of students wearing shoe No. 6?

(iv) Which shoe number is worn by the maximum number of students? Also give its number.

(v) Which shoe number is worn by the minimum number of students? Also give its number.

(vi) State whether true or false:
The number of students wearing shoe No. 10 is less than three times the number of students wearing shoe No. 9.

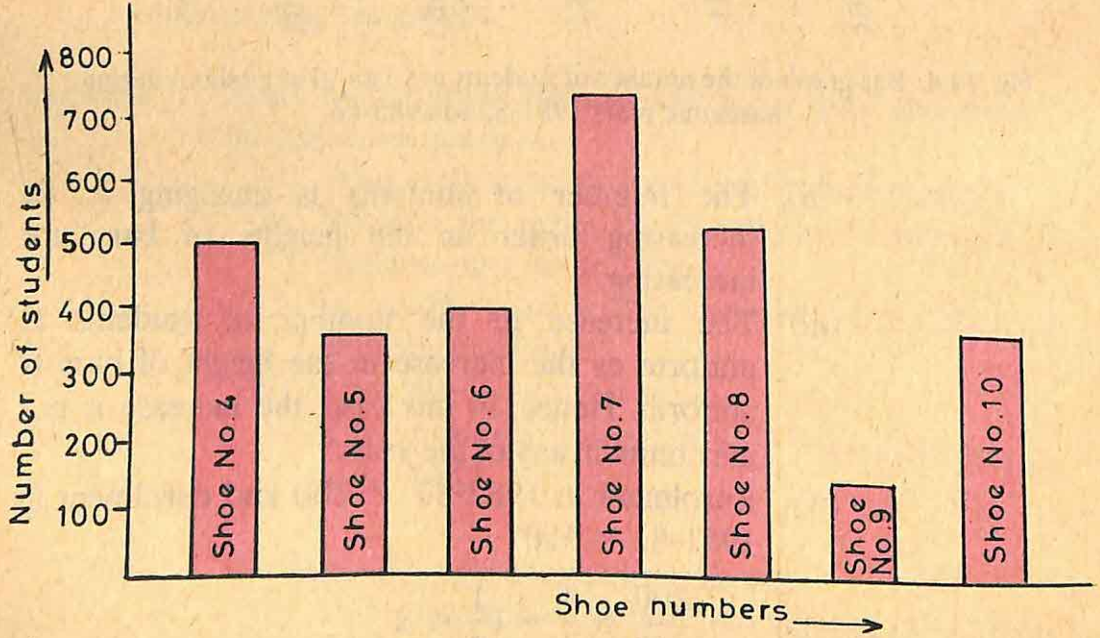


Fig. 14.5: Bar graph of the number of students wearing shoes of different numbers out of a total of 3000 selected students

Solution: The answers to the above questions are:

(i) The bar graph represents the number of students wearing shoes of different numbers out of a total of 3000 selected students.

(ii) The students wear shoes bearing Nos. 4, 5, 6, 7, 8, 9 and 10.

(iii) The number of students wearing shoe No. 6 is 400.

(iv) Shoe No. 7 is worn by the maximum number of students. These students are 725 in number.

(v) Shoe No. 9 is worn by the minimum number of students. These students are 150 in number.

(vi) The number of students wearing shoe No. 10 = 375 and the number of students wearing shoe No. 9 = 150

Therefore, $\frac{375}{150} = \frac{5}{2}$, which is less than 3.

Therefore, the given statement is true.

EXERCISE 14.1

1. Read the bar graph in Fig. 14.6 and answer the following questions:

(i) What is the information given by the bar graph?

(ii) How many tickets of Assam State Lottery were sold by the agent?

(iii) Of which state, were the maximum number of tickets sold?

(iv) Of which state, were the minimum number of tickets sold?

(v) State whether true or false:
The maximum number of tickets sold is three times the minimum number of tickets sold.

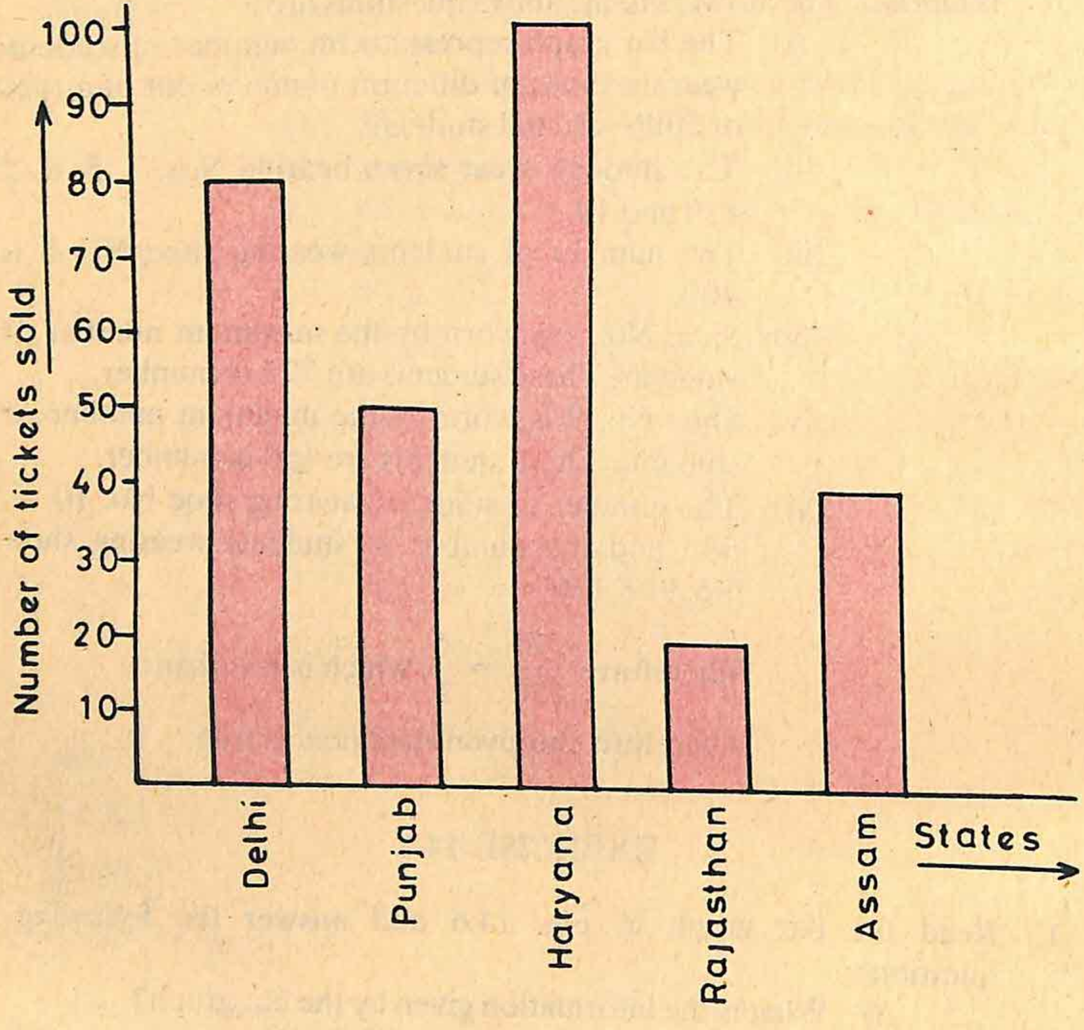


Fig. 14.6: Bar graph of the tickets of different state lotteries sold by an agent on a day

2. Read the bar graph in Fig. 14.7 and answer the following questions:
   - (i) What is the information given by the bar graph?
   - (ii) What is the number of families having 6 members?
   - (iii) How many members per family are there in the maximum number of families? Also tell the number of such families.

(iv) What are the number of members per family for which the number of families are equal? Also tell the number of such families.

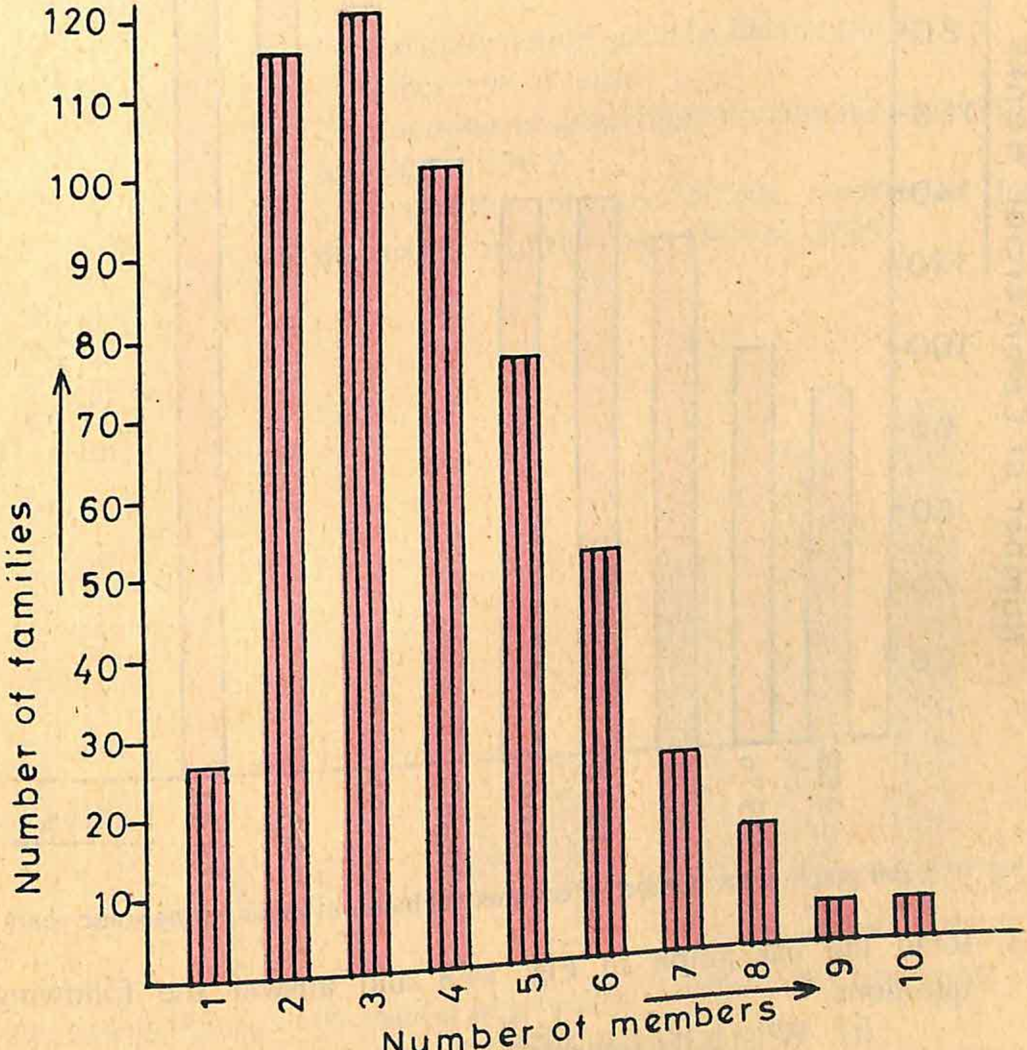


Fig. 14.7: Bar graph of the number of families with different number of members in a locality

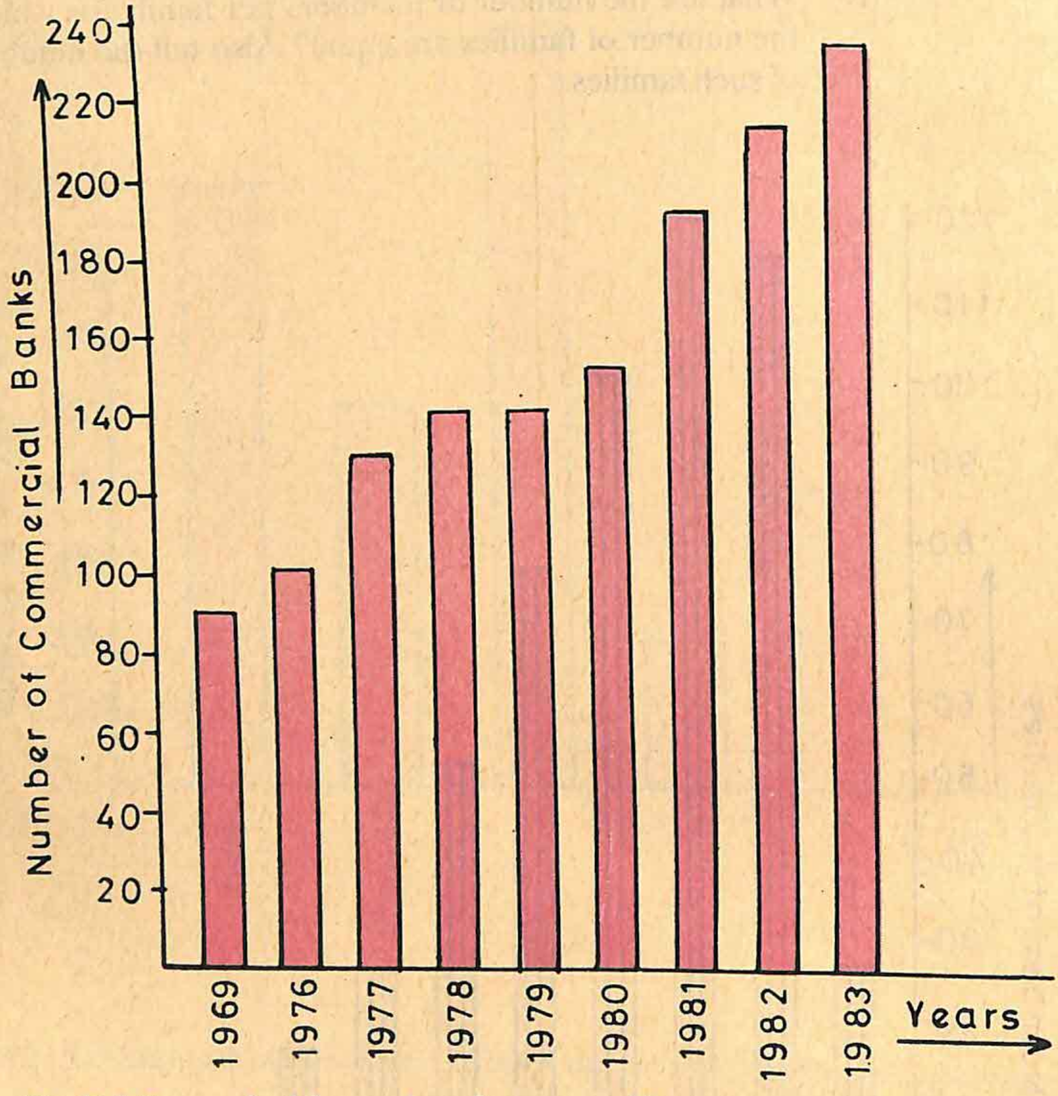


Fig. 14.8: Bar graph of the number of commercial banks in India during some years

3. Read the bar graph in Fig. 14.8 and answer the following questions:
   - (i) What is the information given by the bar graph?
   - (ii) What was the number of commercial banks in 1977?
   - (iii) What is the ratio of the number of commercial banks in 1969 to that in 1980?

(iv) State whether true or false:
The number of commercial banks in 1983 is less than double the number of commercial banks in 1969.

4. Read the bar graph in Fig. 14.9 and answer the following questions:
   (i) What is the information given by the bar graph?
   (ii) State whether true or false:
      (a) The number of government companies in 1957 is half that of 1983.
      (b) The number of government companies have decreased over the years 1957 to 1983.

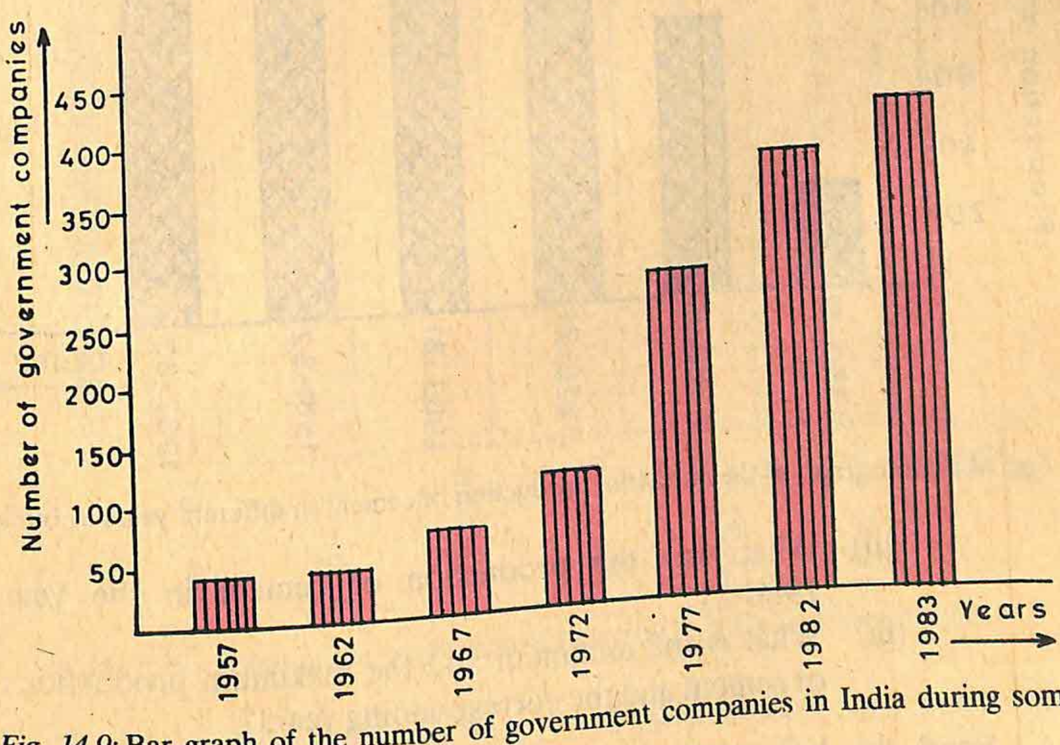


Fig. 14.9: Bar graph of the number of government companies in India during some years

5. Read the bar graph in Fig. 14.10 and answer the following questions:
   (i) What information is given by the bar graph?

(iii) State whether true or false:
The area under the sugarcane crop in the year 1982-83 is three times that of the year 1950-51.

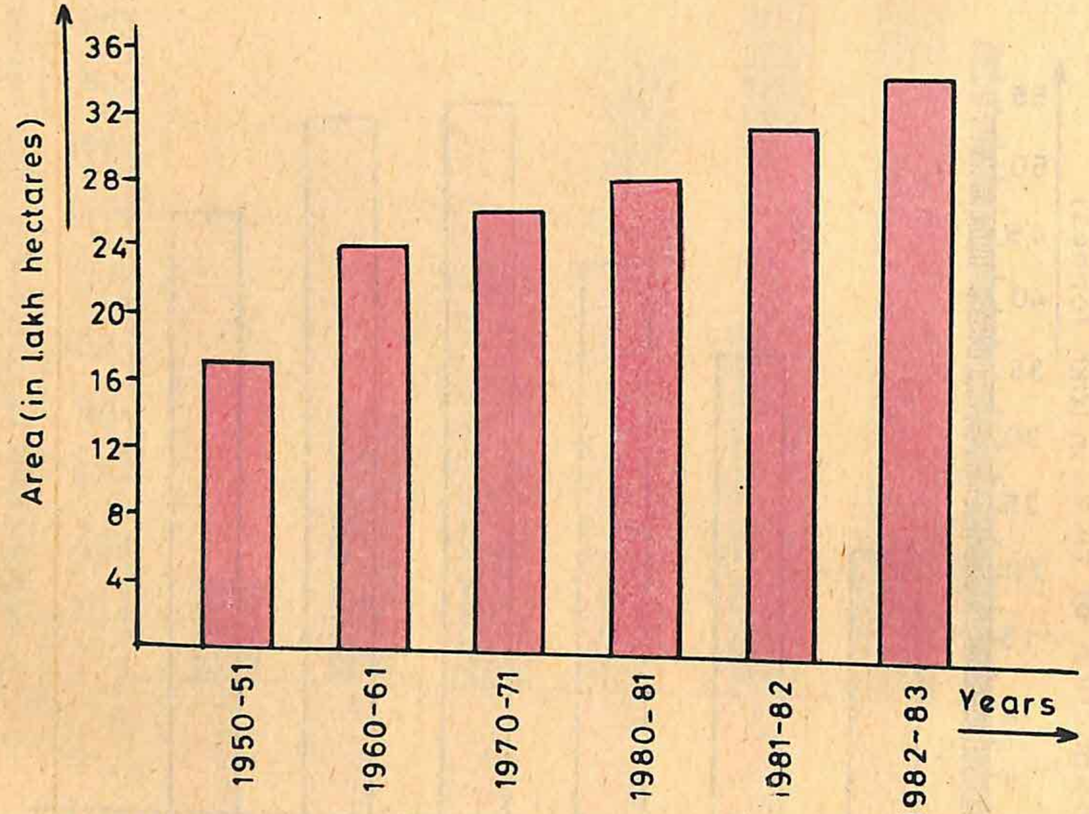


Fig. 14.12: Bar graph of the area under the sugarcane crop during different years in India

8. Read the bar graph in Fig. 14.13 and answer the following questions:
   - (i) What information is given by the bar graph?
   - (ii) What was the expenditure on health and family planning in the year 1982-83?
   - (iii) In which year is the increase in expenditure maximum over the expenditure of the previous year? What is this maximum increase?

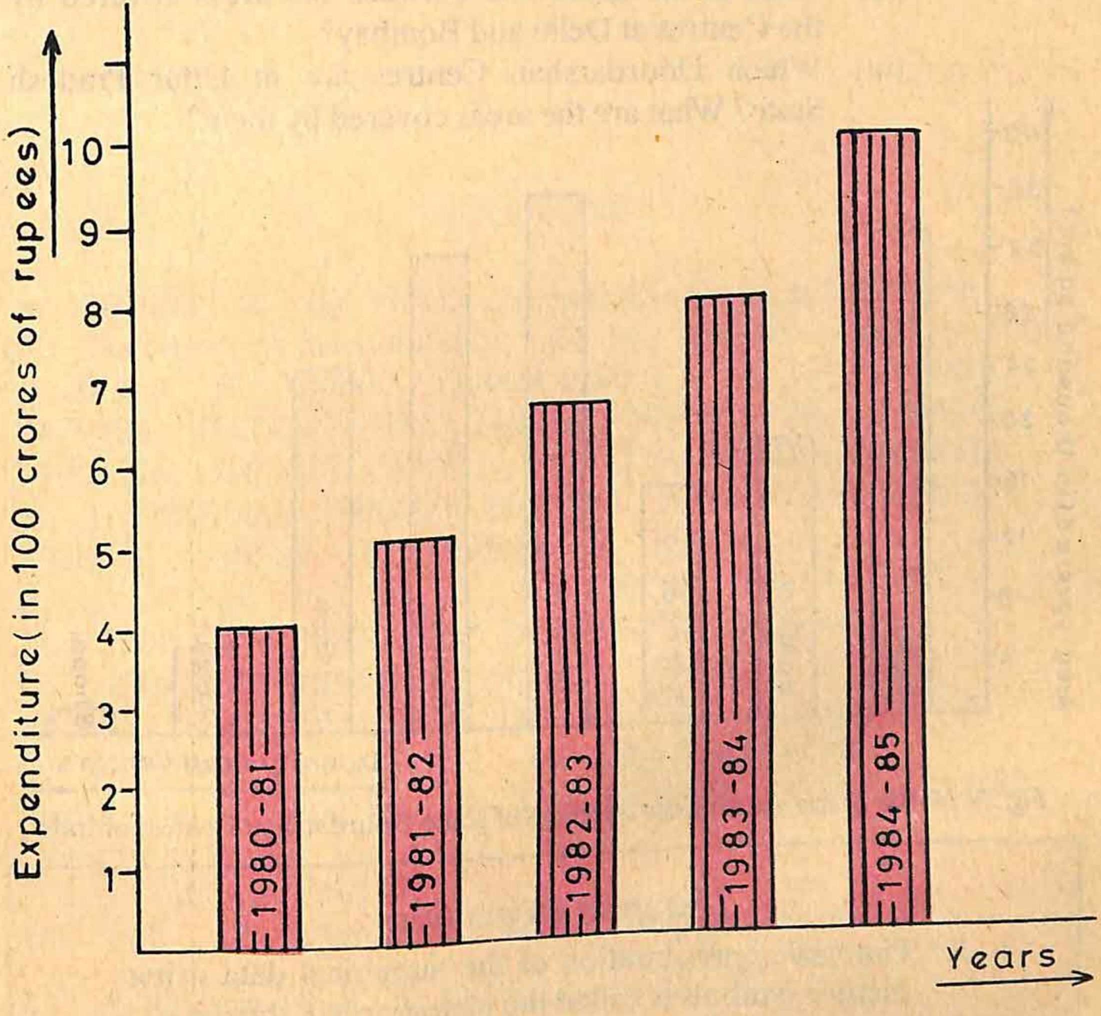


Fig. 14.13: Bar graph of the expenditure on health and family planning during the Sixth Five Year Plan in India

9. Read the bar graph in Fig 14.14 and answer the following questions:
   - (i) What information is given by the bar graph?
   - (ii) Which Doordarshan Centre covers maximum area? Also tell the covered area.

(iii) What is the difference between the areas covered by the Centres at Delhi and Bombay?

(iv) Which Doordarshan Centres are in Uttar Pradesh State? What are the areas covered by them?

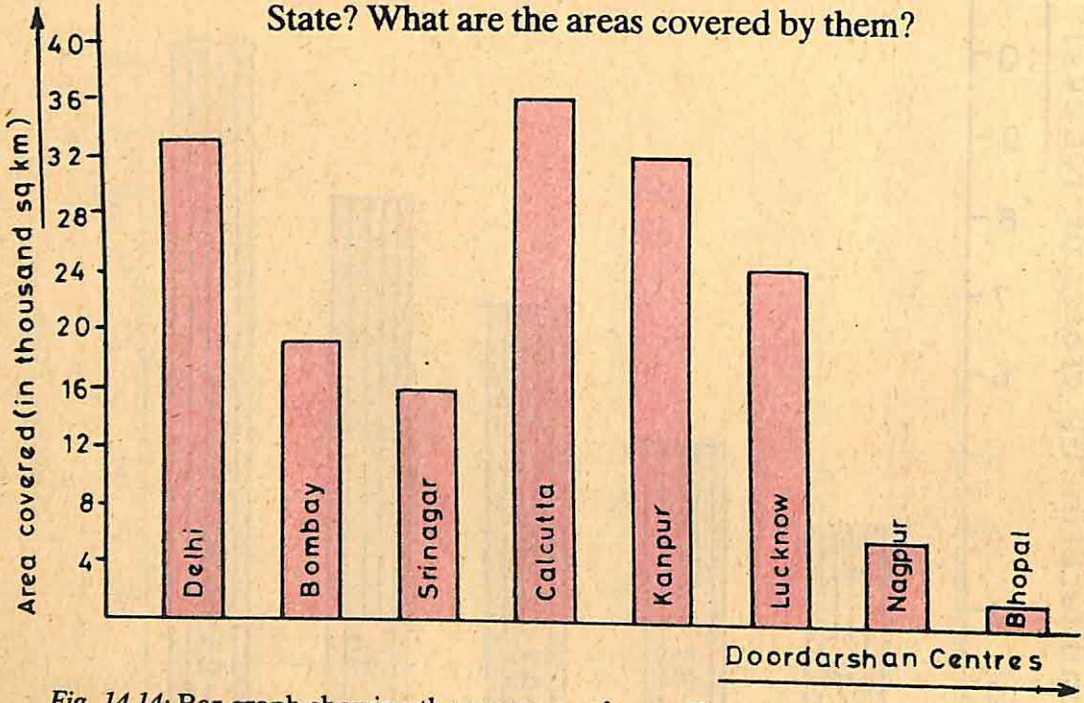


Fig. 14.14: Bar graph showing the coverage of some Doordarshan Centres of India

Things to Remember

1. Pictorial representation of the numerical data using picture symbols is called the pictograph of the data.
2. A bar graph or a bar chart is a pictorial representation of the numerical data by a number of bars of uniform width, erected horizontally or vertically, with equal spacing between them.
3. Bar graphs help in easy comprehension of the given data.
4. Many conclusions can be drawn from a bar graph by just having a glance at it.

Answers

EXERCISE 1

1. 504003 **2.** (i) Ninety-nine lakh five thousand fifty-seven
(ii) Six crore fifty thousand sixty **3.** (i) 600000 + 70000 + 5000
+ 20 + 8 (ii) 700000 + 30000 + 20 + 1 **4.** (i) 7478125,
7479125, 7480125, 7481125, 7482125, 7483125 (ii) 15003216,
15004216, 15005216, 15006216, 15007216, 15008216, 15009216
5. (i) 10000005, 10000000, 9999995 (ii) 5050404, 5050399,
5050394, 5050389 **6.** 499500 **7.** 89999
8. (i) < (ii) < (iii) > **9.** 700000 **10.** 4003999
11. $3000333 < 30030033 < 30303030 < 33003300$
12. $1111010 > 1110202 > 1110011 > 1101011 > 1100111$
13. (i) 336890106 (ii) 0 (iii) 6939380 (iv) 70997766
(v) 68932400 (vi) 451779 **14.** Divisible by 2: (ii);
Divisible by 5: (ii) and (iii) **15.** Divisible by 3: (i) and (iii) and
Divisible by 9: (iii) **16.** (i) 7 (ii) 6
17. (i) 60 (ii) 21450
18. 144 **19.** 27m 72cm **20.** (i) 19 (ii) 22 (iii) 24
21. (i) XIII (ii) XVIII (iii) XXIX **22.** $\frac{1}{3} < \frac{9}{24} < \frac{5}{8} < \frac{5}{6} < \frac{3}{2}$
23. $\frac{8}{9} > \frac{7}{8} > \frac{6}{7} > \frac{15}{18} > \frac{9}{12}$ **24.** (i) $\frac{37}{120}$ (ii) $6\frac{41}{156}$
(iii) 5.3934 (iv) 56.925 **25.** 6.006 **26.** $0.1l$
27. 1250 students **28.** Rs 1.10 **29.** 6 hours 5 minutes
30. (i) 0.04 (ii) 0.4 (iii) 0.75
31. (i) $3\frac{3}{5}$ (ii) $\frac{1}{2}$ (iii) $1\frac{1}{8}$ **32.** 47 km per hour
33. 66m, 270 sq m **34.** 252m, 3969sq m **35.** (i) True
(ii) False (iii) True (iv) True (v) False **36.** 20 days

EXERCISE 3.1

1. (i) 283 (ii) 300507 (iii) 12345
2. No **3.** Yes **4.** (i) 45412 (ii) 923762 (iii) 16112
(iv) 13000 **5.** (i) 1908 (ii) 4400

6.

22	29	6	13	20
28	10	12	19	21
9	11	18	25	27
15	17	24	26	8
16	23	30	7	14

EXERCISE 3.2

1. (i) 6856 (ii) 1305 (iii) 1235 (iv) 1454545

2. (i) $\begin{array}{r} 876 \\ -239 \\ \hline 637 \end{array}$ (ii) $\begin{array}{r} 5376 \\ -2859 \\ \hline 2517 \end{array}$ (iii) $\begin{array}{r} 6000107 \\ -938978 \\ \hline 5061129 \end{array}$

(iv) $\begin{array}{r} 1000000 \\ -\ 56791 \\ \hline 943209 \\ \hline \end{array}$ **3.** 1 **4.** Rs 19575 **5.** 37400

EXERCISE 3.3

1. (i) 0 (ii) 675 (iii) 3709 (iv) 10 (v) 15
(vi) 6 (vii) 4 (viii) 5 (ix) 4 **2.** (i) 173500

37. 50 litres **38.** (i) (c) (ii) (c) (iii) (d) **39.** 10th day
40. (i) $1234 \times 9 + 5 = 11111$ (ii) $1+2+3+4+3+2+1 = 4 \times 4$
(iii) 4356 (iv) $11111 \times 11111 = 123454321$ **41.** (i) 20884
(ii) 1637514 (iii) 4 (iv) 874 lakhs **42.** (a) (i) 110862013
(ii) 30105260 (iii) 80756753 (b) (ii) and (iii)

EXERCISE 2.1

1. (i) 0 (ii) 1, No **2.** (i) 4000000 (ii) 40000
3. 100 **4.** 2500000 **5.** 19998 **6.** (i) 9000 (ii) 900000
7. 90 times **8.** (i) 3000 + 50 + 7 (ii) 200000 + 30000 +
5000 + 60 **9.** (i) 43056119 (ii) 7080077 **10.** Four
11. 101 **12.** 1000 **13.** 1000

14. (i) Five million three-hundred-forty-two thousand seven hundred sixteen (ii) Ten million twenty-three thousand seventy-eight

15. (i) Fourteen crore eighty-eight lakh kilometres (ii) In 1971: Fifty-four crore sixty-nine lakh fifty-five thousand nine hundred forty-five. In 1981 : Sixty-eight crore thirty-eight lakh ten thousand fifty-one

16. One crore seventy-five thousand three hundred two, 10002357
17. 9999987 **18.** 1000023

EXERCISE 2.2

1. (i) 1000910 (ii) 2340701 (iii) 1040000 **2.** (i) 92
(ii) 1999 (iii) 7007999 **3.** 19 **4.** Yes, No **5.** (i) 503
(ii) 370 (iii) 9876 (iv) 100023001 **6.** 1009999, 1010000,
1010001 **7.** 8509998, 8509999, 8510000

8. For any natural number we have a natural number one more than it.

9. (ii) and (iii)

(ii) 16600 (iii) 291000 (iv) 2790000 (v) 85500
(vi) 1000000 **3.** Yes, Yes
4. Yes **5.** No **6.** Yes, 1 **7.** Atleast one of the numbers must be zero **8.** (i) 607920 (ii) 1245616 (iii) 10172486
(iv) 104023689 **9.** (i) 75808 (ii) 81984 (iii) 260064
(iv) 157210 **10.** (i) 2970 (ii) 5427900 (iii) 819500
(iv) 156250000 (v) 19225000 (vi) 887000 (vii) 5790
(viii) 62076 (ix) 461000 (x) 325897 **11.** 999900
12. Rs 1227500 **13.** Rs 18800

EXERCISE 3.4

1. Yes, 1 **2.** No **3.** (i) Quotient 134, Remainder 0
(ii) Quotient 197, Remainder 11 (iii) Quotient 393, Remainder 39
(iv) Quotient 12, Remainder 645 (v) Quotient 16, Remainder 25
(vi) Quotient 309, Remainder 145
4. (i) 32475 (ii) 0 (iii) 486 (iv) 693 (v) 0 (vi) 138
(vii) 1 (viii) 800 **5.** 10 **6.** 35 **7.** 100050 **8.** 9960
9. 718 **10.** 30 trees **11.** 32198 persons **12.** 17 **13.** Rs 780
14. (ii), (iii) and (iv)

EXERCISE 4.1

1. (i) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 (ii) 1, 2, 4, 8, 16, 32, 64
(iii) 1, 2, 4, 19, 38, 76 (iv) 1, 5, 25, 125 (v) 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144 (vi) 1, 3, 9, 27, 81, 243, 729
2. (i) 16, 32, 48, 64, 80 (ii) 17, 34, 51, 68, 85 (iii) 19, 38, 57, 76, 95
(iv) 20, 40, 60, 80, 100 (v) 25, 50, 75, 100, 125
(vi) 40, 80, 120, 160, 200 **3.** (ii) and (iii) **4.** (i) and (iii)
5. (i) 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 (ii) 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157. (iii) 163, 167, 173, 179, 181, 191, 193, 197, 199 **6.** Only one, 2 **7.** (i), (iii) and (v) **8.** Yes, 9 **9.** 90, 91, 92, 93, 94, 95, 96 **10.** Composite

11. 1, 3, 7, 9 **12.** (i) No (ii) Four; 4, 9, 25, 49 **13.** 28
14. 101, 103; 107, 109; 137, 139. **15.** (i) 3 + 29 (ii) 3 + 37
(iii) 3 + 53 (iv) 7 + 73 (v) 7 + 89 (vi) 3 + 97

EXERCISE 4.2

1. Divisible by 2 : (i), (ii), (iii), (iv) and (v); Divisible by 3 : (i), (ii), (iii), (v) and (vi); Divisible by 5: (iii) and (iv); Divisible by 9: (i) and (iii)
2. Divisible by 4: (i), (iii), (iv), (v) and (vi); Divisible by 8: (i) and (iii)
3. (ii) and (iii) **4.** (i), (ii), (iii), (v) and (vi)
5. (i) 11, Yes (ii) 13, No (iii) 13, Yes **6.** (ii), (iii), (v), (vii) and (ix)

EXERCISE 4.3

1. (i) 2 × 2 × 2 × 2 × 3 (ii) 2 × 17 (iii) 2 × 7 × 7
(iv) 2 × 2 × 2 × 3 × 3 × 3 (v) 3 × 5 × 5 × 7
(vi) 2 × 2 × 3 × 3 × 13 (vii) 3 × 3 × 7 × 7
(viii) 2 × 2 × 3 × 3 × 3 × 5 (ix) 2 × 2 × 2 × 3 × 3 × 5 × 5 × 5
(x) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 (xi) 3 × 5 × 11 × 13
(xii) 5 × 5 × 293
2. 1 and the number itself **3.** 10000, 2 × 2 × 2 × 2 × 5 × 5 × 5 × 5
4. 9999, 3 × 3 × 11 × 101 **5.** 7, 13, 19; Difference between them is 6.

EXERCISE 4.4

1. (i) 18 (ii) 9 (iii) 1 (iv) 225 (v) 13 (vi) 10
(vii) 12 (viii) 53 (ix) 1 (x) 625
2. (i) 150 (ii) 13 (iii) 36 (iv) 55 (v) 58 (vi) 747
3. 65583, 65637 **4.** 1 **5.** (i) $\frac{361}{64}$ (ii) $\frac{11}{18}$ (iii) $\frac{11}{25}$ **6.** 16
7. 170 *l* **8.** 17 **9.** 75 cm **10.** 5460 stones

EXERCISE 4.5

1. (i) 240 (ii) 1386 (iii) 180 (iv) 90 (v) 3465 (vi) 90 (vii) 5760 (viii) 18480 (ix) 1620 (x) 6384 **3.** (i) 4095 (ii) 945 **4.** 400 **5.** No **6.** 221 **7.** No, because H.C.F. must also be a factor of L.C.M. **8.** 288 books **9.** 3300 m **10.** 607 **11.** 20 days **12.** 122 m 40 cm **13.** 10080, 9660 **14.** 10080

EXERCISE 5.1

1. (i) Decrease in population (ii) Withdrawing money from bank (iii) Spending money (iv) Going west (v) -12 (vi) $+2$

2. (i) $+3$ (ii) -5 (iii) -25 (iv) 100

3. (i) 7 (ii) -2 (iii) 0 (iv) 8

4. (i) 7 (ii) -3 (iii) -1 (iv) -10

5. (i) -8 (ii) -12 (iii) -15 (iv) -356

6. (i) $-4, -3, -2, -1, 0, 1$ (ii) 1, 2, 3 (iii) $-3, -2, -1, 0, 1, 2, 3$ (iv) $-6, -5, -4, -3, -2, -1$ **7.** (i) < (ii) > (iii) < (iv) < (v) < (vi) > **8.** (i) 17 (ii) 23 (iii) 0 (iv) 107 (v) 245 (vi) 1024 (vii) $x-2$ (viii) $2-x$ **9.** (ii) and (iv)

EXERCISE 5.2

2. (i) -134 (ii) 2564 (iii) 9999 (iv) -98645 (v) -818 (vi) -5894 (vii) 2004 (viii) 0 (ix) -1 (x) -5832 (xi) -1100 (xii) 4690 **3.** (i) 0 (ii) 500 (iii) 600 (iv) -481 (v) 2900 (vi) 1 **4.** (i) -2 (ii) 13 **5.** (i)

EXERCISE 5.3

1. (i) 11 (ii) -14 (iii) 25 (iv) 100 (v) -900 (vi) -9 (vii) 3938 (viii) -8656 (ix) -122 (x) 155 (xi) -1005 (xii) -42920 **2.** 12, -12, No **3.** (i) < (ii) > (iii) >

4. (i) 6 (ii) −19 (iii) 0 (iv) 16 (v) −140 (vi) 151 **5.** 72 **6.** −460 **7.** (i) −4 (ii) 10 (iii) −2 (iv) 0 (v) 100 (vi) −105 (vii) 6 (viii) 8 **8.** −1 **9.** (i) 1 (ii) 1 (iii) −9 (iv) 13 **10.** (ii), (iii) and (iv) **11.** −10 **12.** (i) 2 (ii) 0

EXERCISE 5.4

1. (i) −30 (ii) −1800 (iii) 340 (iv) −120
(v) 162 (vi) −1728 (vii) 360 (viii) 0
(ix) 13320 (x) 24750 (xi) −120 (xii) 1944

2.

Second Number

First Number

×	−4	−3	−2	−1	0	1	2	3	4
−4	16	12	8	4	0	−4	−8	−12	−16
−3	12	9	6	3	0	−3	−6	−9	−12
−2	8	6	4	2	0	−2	−4	−6	−8
−1	4	3	2	1	0	−1	−2	−3	−4
0	0	0	0	0	0	0	0	0	0
1	−4	−3	−2	−1	0	1	2	3	4
2	−8	−6	−4	−2	0	2	4	6	8
3	−12	−9	−6	−3	0	3	6	9	12
4	−16	−12	−8	−4	0	4	8	12	16

Yes, the table is symmetrical about the diagonal.

3. (i) 0 (ii) 887000 (iii) 18300 (iv) 1894600
(v) −1562500 (vi) −480 **4.** (i) Positive (ii) Negative
5. (i) 40 (ii) −46 (iii) 0 **6.** (i) First
(ii) First (iii) First **7.** (i)

EXERCISE 5.5

1. (i) −6 (ii) −6 (iii) 6 (iv) −4

(v) 3 (vi) 0 (vii) −144 (viii) 125
(ix) 9 (x) −10569 (xi) −2000 (xii) −1
2. (i) 1 (ii) −3785 (iii) 0 (iv) −3065
(v) −312 (vi) −567

EXERCISE 5.6

1. (i) 29 (ii) 110 (iii) 0 (iv) 0 (v) 0 (vi) 3 (vii) −3 (viii) 0 (ix) 26 (x) 2 (xi) 36 (xii) −20 **2.** (i) $5 \times (2 + 3)$ (ii) $12 \div (1 + 3)$ (iii) $20 \div (7 - 2)$ (iv) $(2 \times 3) - 8$ (v) $40 \div \{1 + (9 + 10)\}$ (vi) $2 \times \{(19 - 6) - 1\}$ **3.** (i) 29 (ii) 0 (iii) 11 (iv) 87 (v) 119 (vi) 71 (vii) 0 (viii) 109 (ix) −218 (x) 15

EXERCISE 5.7

1. (i) 5, 7 (ii) −2, 3 (iii) 1, 1 (iv) −6, 1 (v) −27, 2 (vi) 10, 5 **2.** (i) 10^4 (ii) $(-13)^6$ **3.** (i) 2500 (ii) −1 (iii) 1 (iv) 1 (v) 256 (vi) 72 (vii) 256 (viii) 16 (ix) −64 (x) 432 (xi) −100 (xii) 576
4. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. The digits 2, 3, 7, 8 do not occur in unit's place. **5.** 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000 **6.** (i) 400 (ii) 10000 (iii) 40000 (iv) 4900 (v) 22500 (vi) 1000000 **7.** (i) −1728 (ii) −2197 (iii) −3375 (iv) 1331 (v) 1000000 (vi) 1000000000 **9.** (ii), (iii), (iv) and (v)

EXERCISE 6.1

1. (i) $d = 2r$; d = diameter, r = radius (ii) $A = l \times b$; A = Area, l = length; b = breadth; (iii) $s = c + p$; s = selling price; c = cost price, p = profit **2.** (i) $6 + x$ (ii) $y + 3$ (iii) $\frac{x}{3}$ (iv) $\frac{x + y}{2}$ (v) $7 - y$ (vi) $x - 7$ (vii) $\frac{x}{y} - 2$

EXERCISE 6.2

1. a^{10} (ii) $17x^3y^3$ (iii) y^{20} (iv) $7a^2b^5c$
2. (i) $a \times a \times b \times b \times b \times b \times b$ (ii) $8 \times z \times z \times z$
(iii) $9 \times a \times b \times b \times b$ (iv) $10 \times x \times x \times x \times y \times y \times y \times z \times z \times z$
3. xy^2

EXERCISE 6.3

1. (i) $3x^5, 5y^4, -7x^2y, 7$ (ii) $9y^3, -2z^5, 7x^3y, -3xyz$ (iii) $a^5, -3ab, -b^2, 6$ **2.** Monomials: (ii), (vi) and (viii); Binomials: (i) and (v); Trinomials: (iii) and (iv)
3. (i) 3 (ii) $-4a$ (iii) $5y^2$ (iv) yz **4.** $1, -7, 5, -2$ **5.** x^2, x and x^3 **6.** (i) $x^2, 2x^2$ (ii) $yz, \frac{zy}{2}$ (iii) $-2x^2y, -yx^2$ (iv) cab^2, b^2ac, ab^2c, acb^2 **7.** (i) 14 (ii) 39 (iii) 27 (iv) 76 (v) 23 (vi) -1

EXERCISE 6.4

1. (i) $-4x^3$ (ii) $-x^2y$ (iii) $4xyz$ **2.** (i) $8a^5$ (ii) $-18ab$
(iii) $-3x^2$ (iv) $-8x^2y$ (v) $7x^2$ (vi) $-3z^2b - 9a^2b$ **3.** (i) $2a + 3b + 2c$
(ii) $3x + 2y + 2z$ (iii) $3x^2 + 4y^2 + 3$ (iv) 0 (v) $3a^2 + b^2$
4. (i) $-2x^2 - 3y^2 + 2$ (ii) $-5xy + 3yz - 3zx$ **5.** (i) $-3a + 4b - 2$
(ii) $6x^2 - y + 4z + 7$ (iii) $6a - 3b - c - 4d$ **6.** $20r^2s^2 - 11rs + 14$
7. $-a^2 + 2ab$ **8.** $-11x + y - 5z - 7$
9. $5a^2 + 3b^2 - 7ab + 6 - a$

EXERCISE 6.5

1. (i) $4ab$ (ii) $3a - 7b - 7$ (iii) $-3x^2 - 2y^2 - 4$ (iv) $3x^2z +$
$3xy - 3yz + 7z$ (v) $-2n$ (vi) $a + a^2 + 3b - 2c$
(vii) $104x + 44$ (viii) $-12x^3 + 5x^2 + 19x - 4$ (ix) $xy + 2xz -$
$3y$ (x) $1 + 7x + y - 3x^2$ **2.** (i) $9a + 5xy - 7x^2 - (-8y + 6)$
(ii) $-y + z + x^2 - (y^2 + a^2)$ (iii) $x + y + z - xy - (yz + zx)$
(iv) $xy^2 - (-yz^2 - zx^2)$

EXERCISE 7.1

1. (i) L.H.S. $x - 2$, R.H.S. 2 (ii) L.H.S. $2y$, R.H.S: $9 - y$
(iii) L..H.S. $2p$, R.H.S. 6 (iv) L.H.S. $2x + y$, R.H.S. $7 + z$
2. (i) $2y - 3 = 17$ (ii) $15 - 2y = 7$ (iii) $y^2 = y + 6$
(iv) $\frac{1}{6}y = 7$ (v) $2y + 6 = 30$ (vi) $\frac{2}{3}y = 10$

EXERCISE 7.2

1. (i) 5 (ii) 35 (iii) 6 (iv) 6 (v) -4 (vi) 12 (vii) 72
(viii) 4 (ix) 9

EXERCISE 7.3

1. -12 **2.** $\frac{43}{2}$ **3.** $\frac{7}{2}$ **4.** 15 **5.** 75 **6.** -2

7. $\frac{3}{2}$ **8.** $\frac{7}{3}$ **9.** 4 **10.** $\frac{7}{2}$ **11.** -3 **12.** 15

13. 175 **14.** $\frac{7}{2}$ **15.** 1 **16.** 0

EXERCISE 7.4

1. 4 **2.** 3 **3.** 36 **4.** 14 years, 42 years **5.** 7 metres
6. Boys 604, Girls 656 **7.** 250 **8.** 5 years
9. Men : 12, Women: 36 **10.** 5, 25 **11.** Babita is 6 years,
Anjali is 10 years **12.** Length 100 metres, Width 50 metres
13. 26, 27 **14.** 42, 44 **15.** Rice 9 kg, Wheat 27 kg **16.** A.D. 375

EXERCISE 8.1

1. (i) 6 : 1 (ii) 1 : 2 (iii) 2 : 3 **2.** (i) 3 : 8 (ii) 1 : 3 (iii) 3 : 5
(iv) 9 : 19 **3.** (i) 140 : 3 (ii) 3 : 10 (iii) 20 : 1 (iv) 240 : 1

4. (i) 1 : 8 (ii) 8 : 1 **5.** (i) 4 : 5 (ii) 4 : 9 **6.** (i) 37 : 191 (ii) 191 : 154 (iii) 37 : 154 **7.** (i) 2 : 35 (ii) 40 : 1 (iii) 1 : 240 (iv) 3 : 5 (v) 40 : 1 (vi) 5 : 1 **8.** (i) 7 : 11 (ii) 7 : 18 (iii) 18 : 11 **9.** 2 : 15 **10.** 3 : 350

EXERCISE 8.2

1. (i) True (ii) True (iii) False (iv) True (v) True **2.** (i), (iv), (vi) **3.** $\frac{25}{3}$ **4.** (i) 30 (ii) 2 (iii) 30 (iv) $\frac{256}{3}$ **5.** 9 : 150 = 105 : 1750, 9 : 105 = 150 : 1750, 105 : 9 = 1750 : 150, 150 : 9 = 1750 : 105 **6.** (i) 45 : 24 = 30 : 16, 30 : 45 = 16 : 24, 24 : 45 = 16 : 30 (ii) 12 : 14 = 18 : 21, 18 : 12 = 21 : 14, 14 : 12 = 21 : 18 **7.** 100 metres **8.** Rs 80 **9.** 675 **10.** 12.6 kg **11.** 49 **12.** Rs 200 **13.** 4 km

EXERCISE 8.3

1. Rs 240 **2.** Rs 910 **3.** 845 kg **4.** (i) 8 hours (ii) 357.5 km **5.** 10000 chairs **6.** 3 hours **7.** $4\frac{23}{28}$ hectares **8.** 100 **9.** (i) 10 kg (ii) 48 **10.** Rs 397.50 **11.** Rs 5.40, 300

EXERCISE 9.1

1. (i) 9 : 50 (ii) 1 : 8 (iii) 7 : 5 (iv) 1 : 16 (v) 1 : 50 (vi) 3 : 1000 **2.** (i) $\frac{1}{6}$ (ii) $\frac{9}{20}$ (iii) $\frac{3}{2}$ (iv) $\frac{1}{400}$ **3.** (i) 0.125 (ii) 0.75 (iii) 0.015 (iv) 1.288 (v) .006 (vi) .07 **4.** (i) 25 % (ii) $71\frac{3}{7}$% (iii) 37.5 % (iv) 525 % (v) 0.5 % (vi) 245 % (vii) 45 % (viii) 1640% (ix) $187\frac{1}{2}$% **5.** Gita **6.** (i) 300 (ii) 4 litres (iii) 24 kg (iv) Rs 81 (v) 35 km (vi) 42

7. (i) 25 % (ii) $16\frac{2}{3}$% (iii) 120 % (iv) 40 % (v) 30 % (vi) 40 %
8. 28845 **9.** 126 **10.** 714000000 **11.** $66\frac{2}{3}$% **12.** 40 %

13. 32 % **14.** $12\frac{1}{2}$% **15.** 288 **16.** 484 **17.** 12 %

18. $85\frac{5}{7}$% **19.** $33\frac{1}{3}$%

EXERCISE 9.2

1. (i) Profit = Rs 80 (ii) S.P. = Rs 428 (iii) C.P. = Rs 9900 (iv) Loss = Rs 30 **2.** (i) S.P. = Rs 540; Profit % = 20 (ii) S.P. = Rs 3038; Loss % = 2 (iii) Profit = Rs 6000; Profit % = 20 (iv) S.P. = Rs 972; Profit = Rs 72 (v) S.P. = Rs 235; Loss = Rs 15 **3.** 35 % profit **4.** Rs 3.60 **5.** Rs 4140 **6.** Rs 2350
7. Rs 80.50 **8.** Rs 1450 **9.** 46 % gain **10.** $\frac{5}{16}$ % loss
11. 40 paise

EXERCISE 9.3

1. (i) Rs 180 (ii) Rs 700 (iii) Rs 60, Rs 260 (iv) Rs 210, Rs 810 **2.** Rs 1050 **3.** Rs 12060 **4.** Rs 1920 **5.** Rs 83.20, Rs 603.20 **6.** Rs. 28.80 **7.** Rs 572 **8.** Rs 40

EXERCISE 10.1

4. Line AB, Line m, Line PQ **7.** Unlimited number of lines
8. One line **9.** No **10.** Three lines
11. (i) Collinear (ii) Non-collinear (iii) Collinear (iv) Non-collinear
12. (i) l, m; m, n; l, n (ii) l, p; m, p; n, p; l, r; m, r; n, r; p, r; l, q; m, q; n, q; q, p; q, r (iii) m, p (iv) l, r (v) m, r (vi) l, q

(vii) G, A, B, C; D, E, J, F; G, H, I, J, K; A, H, D; B, I, E; C, F, K
14. One **15.** n, q, l; point of concurrence A. q, m, p; point of concurrence B. **16.** (i) Six lines (ii) Lines AB, BC, CD, AD, BD, AC (iii) Lines AC, AB, AD **17.** Three, Zero
18. Six, Zero **19.** Concurrent **20.** No, No **21.** (i) F (ii) T (iii) F (iv) F (v) F (vi) F (vii) F (viii) F (ix) T (x) T (xi) F

EXERCISE 11.1

1. (i) Line-segments AB, BC, AC; three (ii) Line-segments AB, AD, BD, DC, BC, AC; six (iii) Line-segments AB, BC, AC, AD, DC, BD, BE, DE; eight (iv) Line-segments AC, AB, BC, DE, EF, DF; six
(v) Line-segments PS, SR, RQ, PQ, PR, PO, OR, SQ, SO, OQ; ten

EXERCISE 11.2

2. (i) 1400 cm (ii) 706 cm (iii) 506 cm
(iv) 580 cm (v) 106.7 cm (vi) 70 cm (vii) 9 cm (viii) 8.3 cm (ix) 267 cm
3. (i) 70 mm (ii) 45 mm (iii) 8630 mm **4.** 2.7 cm
7. (i) BC = 4.5 cm, BD = 1.4 cm (ii) PQ = 4.7 cm, OR = 1.3 cm
8. (i) No (ii) Yes **13.** 7.3 cm

EXERCISE 12.1

1. A ray has one end-point and a line has none.
3. (i) Rays AP, AQ, AB, AC, BP, BA, BQ, BC, CP, CQ, CA, CB
(ii) No (iii) Yes (iv) Yes (v) No **5.** Four; $\angle ABC$, $\angle BCD$, $\angle ADC$, $\angle BAD$ **6.** Eight; $\angle ADC$, $\angle CAD$, $\angle ACD$, $\angle CAB$, $\angle ABC$, $\angle ACB$, $\angle BAD$, $\angle BCD$; two **7.** (i) J, C (ii) B, D (iii) A, P, M
8. Vertex M; arms ML and MP **9.** (i) $\angle BOD$ or $\angle DOB$ (ii) $\angle BOC$ or $\angle COB$ (iii) $\angle AOC$ or $\angle COA$ (iv) $\angle AOD$ or $\angle DOA$ **10.** (i) No
(ii) Yes (iii) Yes (iv) Yes (v) No

EXERCISE 12.2

1. (i) ∠1 (ii) ∠a (iii) ∠ABC (iv) ∠3 (v) ∠AOB **2.** (a) (ii), (v) (b) Yes (c) No (d) Yes (e) No (f) No **5.** Acute angle **6.** (i) North (ii) West (iii) South (iv) East **7.** (i) South-West (ii) North-East **8.** (i) Obtuse (ii) Acute (iii) Obtuse (iv) Zero (v) Acute (vi) Straight (vii) Acute (viii) Acute (ix) Right (x) Obtuse (xi) Obtuse (xii) Complete **9.** (i) Straight angle (ii) Right angle (iii) Acute angle (iv) Obtuse angle **10.** (i) Acute (ii) Obtuse (iii) Straight (iv) Right (v) Complete

EXERCISE 12.3

1. ∠DOC, ∠COB; ∠COB, ∠BOA; ∠DOB, ∠AOB; ∠DOC, ∠AOC **2.** (i) Yes (ii) Yes (iii) No (iv) No **3.** (i) ∠1, ∠2; ∠2, ∠4; ∠3, ∠4; ∠1, ∠3; ∠5, ∠6; ∠6, ∠7; ∠7, ∠8; ∠5, ∠8 (ii) ∠1, ∠4; ∠2, ∠3; ∠5, ∠7; ∠6, ∠8 **4.** No **5.** (i) 110° (ii) 45° (iii) 130° (iv) 60° (v) 90° **6.** (i) 35° (ii) 17° (iii) 45° (iv) 50° (v) 60° **7.** (i) Complementary (ii) Supplementary (iii) Complementary (iv) Complementary (v) Supplementary (vi) Supplementary (vii) Complementary **8.** Right angle **9.** Angle of 45° **10.** Obtuse **11.** Acute **12.** Right angle **13.** (i) No (ii) Yes (iii) No **14.** No **15.** Less than 45° **16.** (i) Yes (ii) No (iii) Yes **17.** (i) Yes (ii) Yes **18.** (i) T (ii) T (iii) F (iv) F (v) F (vi) T

EXERCISE 12.4

1. (i) 60° (ii) 150° **4.** Yes, Yes **7.** (i) ∠a (ii) ∠1 (iii) ∠q **8.** (i) Obtuse (ii) Acute (iii) Right (iv) Right (v) Obtuse (vi) Acute **9.** Pair of angles in (iii) are equal

EXERCISE 12.6

1. (i) One (ii) One **3.** (i) Not perpendicular (ii) Perpendicular (iii) Perpendicular

EXERCISE 13.1

1. (i) $DE \parallel BC$ (ii) $AB \parallel CD, AD \parallel BC$ (iii) $PO \parallel TS, UP \parallel SR, UT \parallel QR$ (iv) $AC \parallel FE, BC \parallel FD, AB \parallel DE$

3. AB is not parallel to CD because AB and CD intersect each other when extended.

EXERCISE 13.2

(i) Alternate angles: $\angle BGH$ and $\angle CHG$; $\angle AGH$ and $\angle DHG$ Corresponding angles: $\angle EGB$ and $\angle GHD$; $\angle EGA$ and $\angle GHC$; $\angle BGH$ and $\angle DHF$; $\angle AGH$ and $\angle CHF$ (ii) $\angle e, \angle c, \angle b, \angle a$ (iii) $\angle QRA, \angle BRA, \angle BRA$ (iv) Interior angles: $\angle d, \angle f$; $\angle a, \angle e$; Exterior angles: $\angle c, \angle g$; $\angle b, \angle h$

EXERCISE 13.3

1. $\angle QMD = 120^\circ$, $\angle PLB = 60^\circ$, $\angle ALM = 60^\circ$, $\angle MLB = 120^\circ$, $\angle DML = 60^\circ$, $\angle CML = 120^\circ$, $\angle ALP = 120^\circ$

2. $\angle ALM = 35^\circ$, $\angle PLA = 145^\circ$ **3.** (i) 45° (ii) 75° (iii) 60°

EXERCISE 13.4

3. Two

EXERCISE 14.1

1. (i) Number of tickets of different state lotteries sold by an agent on a day (ii) 40 (iii) Haryana (iv) Rajasthan (v) False
2. (i) Number of families with different number of members in a locality (ii) 50 (iii) 3, 120 (iv) 9 and 10, 5
3. (i) Number of commercial banks in India during some years (ii) 130 (iii) 3:5 (iv) False
4. (i) Number of government companies in India during some years (ii) (a) False (b) False
5. (i) Industrial production of cement in different years in India (ii) 186 lakh tonnes (iii) Minimum: 30 lakh tonnes, 1950-51; Maximum: 232 lakh tonnes, 1982-83
6. (i) Production of rice crop in India in different years (ii) 42.5 lakh tonnes (iii) 33 lakh tonnes
7. (i) Areas under the sugarcane crop during different years in India (ii) Maximum: 1982-83; Minimum: 1950-51 (iii) False
8. (i) Expenditure on health and family planning during the Sixth Five Year Plan in India (ii) Rs 700 crores (approx.) (iii) 1984-85, Rs 200 crores
9. (i) Coverage of some Doordarshan Centres of India (ii) Calcutta, 36000 sq km (iii) 14000 sq km (iv) Kanpur, Lucknow; 32000 sq km, 25000 sq km

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